shorter-term contract expires. We take delivery of the bond, financing it at the repo rate, r, and hold it until time T, when the longer-term contract expires. Because we are short that contract, we simply deliver the bond. Assume we can identify today the cheapest bond to be delivered on the shorter-term contract.

Consider the following notation:

$CF(t)$ = conversion factor for bond delivered at t
$CF(T)$ = conversion factor for same bond delivered at T
$f_0(t)$ = today's futures price for contract expiring at t
$f_0(T)$ = today's futures price for contract expiring at T
AI_t = accrued interest on bond as of time t
AI_T = accrued interest on bond as of time T

At time t, we take delivery of the bond and pay the invoice price,

$$(CF(t))f_0(t) + AI_t.$$

To finance the acceptance and holding of this bond, we borrow this amount at the rate r. Then, at time T, we deliver the bond and receive the invoice price plus coupons received and reinvested from t to T,

$$(CF(T))f_0(T) + AI_T + CI_{t,T}.$$

Since this transaction is riskless, the profit from it should be zero. Therefore,

$$[CF(t)\, f_0(t) + AI_t](1 + \hat{r})T^{-1} = (CF(T)f_0(T) + AI_T + CI_{t,T}.$$

The bracketed term on the left-hand side is the amount we paid for the bond at t. Since we borrowed this amount, we must factor it up by the interest rate compounded over the period T − t. The right-hand side is the amount received from delivering the bond at T. We can now solve for , the implied repo rate:

[redacted]

The numerator is the amount received for the bond, and the denominator is the amount paid for it. Dividing these two numbers gives the rate of return over the period T − t. Raising this term to the power 1/(T − t) annualizes the rate. The implied repo thus is the return we could earn over the period T − t. If the bond can be financed at less than this rate, the transaction will be profitable.

An Example Assume that on December 2, 2005, the cheapest bond to deliver was the 6 1/4s maturing on August 15, 2023. Let us examine the March–June Treasury bond futures spread. The March contract is priced at 112, and the conversion factor is 1.0269. The June futures price is 111.75. The conversion factor for the 6 1/4s delivered on the June contract is 1.0265. The accrued interest on the bond on March 7, the assumed delivery date, is 0.35, and the accrued interest on June 5 is 1.90. There are no coupons between the two futures expiration dates so $CI_{t,T} = 0$.

Since the time from March 7 to June 5 is 90 days, the implied repo rate is

$$\hat{r} = \left[\frac{111.75(1.0265)+1.90}{112.00(1.0269)+0.35}\right]^{(365.90)} - 1 = 0.0446.$$

The implied repo rate thus is 4.46 percent. Note that this is a forward rate, because it reflects the repo rate over the future period from March 7 to June 5. If the bond could be financed at a rate of less than 4.46 percent from March 7 to June 5, the transaction would be profitable.

One way traders determine if the implied repo rate on the spread is attractive is to evaluate what is called a turtle trade. The implied repo rate of 4.46 percent is an implied forward rate. It can be compared to the implied rate in the Fed funds futures market. If the Fed funds futures rate is lower, the trader sells the Fed funds futures and buys the T-bond spread. This creates a risk-free position and earns the difference between the implied repo rate on the T-bond spread and the implied rate on the Fed funds futures. If the implied rate on the Fed funds futures is higher, the investor reverses the T-bond spread and buys the Fed funds futures.

STOCK INDEX ARBITRAGE

We discussed carry arbitrage with Federal funds, Eurodollars, and bond futures. The concept is equally applicable to stock index futures. In fact, this type of transaction is one of the most widely used in the futures markets. It is called stock index arbitrage.

Recall that the model for the stock index futures price when interest and dividends are expressed in continuously compounded form is

$$f_0(T) = S_0 e^{(r_c - \delta_c)T},$$

where r_c is the continuously compounded risk-free rate and δ_c is the continuously compounded dividend yield. Consider the following example of a futures contract that has 40 days to go until expiration. The S&P 500 index is at 1305, the risk-free rate is 5.2 percent, and the dividend yield is 3 percent. The time to expiration will be $40/365 = 0.1096$. Thus, the futures should be priced at

$$f_0(T) = 1305e^{(0.052-0.03)(0.1096)} = 1{,}308.15.$$

Now suppose the actual futures price is 1309.66. Thus, the futures contract is slightly overpriced. We would sell the futures and buy the stocks in the S&P 500 index in the same proportions as in the index. At expiration, the futures price would equal the spot price of the S&P 500 index. We then would sell the stocks. The transaction is theoretically riskless and would earn a return in excess of the risk-free rate.

Now suppose at expiration the index closes at 1,300.30. The profit on the futures contract is $1{,}309.66 - 1{,}300.30 = 9.36$. We bought the stocks for 1305; however, over the life of the futures this investment lost interest at a rate of 5.2 percent and accumulated dividends at a rate of 3 percent. Thus, the effective cost of the stock was $1{,}305e^{(0.052-0.03)(0.1096)} = 1{,}308.15$, which by no coincidence is the theoretical futures price. The stock is sold at 1,300.30 for a profit of $1{,}300.30 - 1{,}308.15 = -7.85$. Thus, the overall profit is $9.36 - 7.85 = 1.51$. This is the difference between the theoretical futures price and the actual futures price. Since the actual futures price was higher than the theoretical price we were able to execute an arbitrage involving the purchase of stocks and sale of futures to capture the 1.51 differential. Had the actual futures price been less than the theoretical price, then we would have executed a reverse carry arbitrage involving the purchase of futures and short sale of stock, which would have created a synthetic loan that would have cost less than the risk-free rate.

Let us now determine the implied repo rate. Given the pricing formula, $f_0(T) = S_0 e^{(r_c - \delta_c)T}$, suppose the futures price is equal to its theoretical fair price. Then we solve this equation for the implied interest rate, $\hat{r}_c$, and obtain

$$\hat{r}_c = \frac{\ln(f_0(T)/S_0)}{T} + \delta_c.$$

In this example, we have

$$\hat{r}_c = \frac{\ln(1{,}309.66/1305)}{0.1096} + 0.03 = 0.0625.$$

If this transaction were undertaken, it would provide a risk-free return of 6.25 percent. With the risk-free rate at 5 percent, the transaction is an attractive opportunity. Also, if an arbitrageur could not borrow at 5 percent, but could borrow at any rate less than 6.25 percent, the transaction would still be worth doing.

Stock index arbitrage has proven to be particularly popular. It turns out, however, that there are a number of serious practical considerations that can limit its profitability.

Some Practical Considerations There are several problems in implementing stock index arbitrage. We referred to the arbitrageur as buying the stock index at 1305. In reality, the arbitrageur would have to purchase all 500 stocks in the appropriate proportions as the index and immediately execute all of the trades. The New York Stock Exchange has established a computerized order processing system, called the Designated Order Turnaround, or DOT, that expedites trades. Nonetheless, it is still difficult to get all the trades in before the price of any single stock changes. Thus, most arbitrageurs do not duplicate the index but use a smaller subset of the stocks. Naturally this introduces some risk into what is supposed to be a riskless transaction.[5]

Let us assume, however, that the trades can be executed simultaneously. Let the index be 1305. Now assume an arbitrageur has $20 million to use. Then the arbitrageur will buy the appropriately weighted 500 stocks with that amount. Because of the $250 multiplier on the futures, the S&P 500 is actually priced at 1,305(250) = $326,250, so the arbitrageur will need to buy $20,000,000/$326,250 = 61.30 futures contracts. Because one cannot buy fractional contracts, the transaction will not be weighted precisely.

In addition, there are transaction costs of about 0.5 percent of the market value of the stocks. Would this consume the profit in this example? If the index is 1,305 and the net profit is 1.51, the profit is approximately 0.0012 percent of the index and clearly would be absorbed by the transaction costs.

In addition, there are problems involved in simultaneously selling all of the stocks in the index at expiration. These transactions must be executed such that the portfolio will be liquidated at the closing values of each stock. This is very difficult to do and frequently causes unusual stock price movements at expiration.

Nonetheless, many large financial institutions execute this type of arbitrage transaction. Every day billions of dollars trade on the basis of this futures pricing model. This trading of large blocks of stock simultaneously is called program trading.[6] The New York Stock Exchange defines a program trade as the simultaneous or near simultaneous purchase or sale of at least 15 stocks with a total market value of at least $1 million. The NYSE requires that these program trades be reported to it and all index arbitrage trades must also be reported.

In this type of trading, the model is programmed into a computer, which continuously monitors the futures price and the individual stock prices. When the computer identifies a deviation from the model, it sends a signal to the user. Many large institutions have established procedures for immediately executing the many simultaneous transactions, usually sending the orders through the DOT system. Table 10.5 illustrates a successful index arbitrage trade using the same example as before, but with a more significantly overpriced futures.

Are stock index futures contracts correctly priced? Is it truly possible to profit from index arbitrage? This question has been studied at great length. In the early days of stock index futures trading, there was considerable evidence that stock index futures prices were too low (Figlewski, 1984). In time, prices began to conform more closely to the model, as shown by Cornell (1985). Deviations from the model remain, however, and some can be exploited by traders with sufficiently low transaction costs. MacKinlay and Ramaswamy (1988) revealed that (1) mispricing is more common the longer the remaining time to expiration and (2) when a contract becomes overpriced or underpriced, it tends to stay overpriced or

[5]As an alternative, the arbitrageur could use exchange-traded funds, which are securities that represent claims on a portfolio identical to an underlying index. Many exchange-traded funds are very actively traded.

[6]Index arbitrage is but one form of program trading. Another is portfolio insurance, which is covered in Chapter 14. See Hill and Jones (1988) for a discussion of the different forms of program trading.

underpriced rather than reversing from overpriced to underpriced or vice versa. Sofianos (1993) found that it was very difficult to profit from index arbitrage after accounting for the problem of simultaneously executing all trades.

One consequence of program trading is that large stock price movements often occur quickly and without an apparent flow of new information. For example, when the index or futures price becomes out of line with the carry model, many investors recognize this event simultaneously and react by buying and

Table 10.5 Stock Index Arbitrage

Scenario: On November 8, the S&P 500 index is at 1,305; the continuously compounded dividend yield is 3 percent; and the continuously compounded risk-free rate is 5.2 percent. The December futures contract, which expires in 40 days, is priced at 1316.30. Its theoretical price is

$$f = 1{,}305e^{(0.052-0.03)(0.1096)} = 1{,}308.15,$$

where T = 40/365 = 0.1096. Thus, the futures contract is overpriced and the carry arbitrage transaction will be executed using $20 million. Transaction costs are 0.5 percent of the dollars invested.

Date	Spot Market	Futures Market
November 8	The S&P 500 is at 1,305. The stocks have a dividend yield of 3 percent. The risk-free rate is 5.2 percent. **Buy $20 million of stock in the same proportions as make up the S&P 500**	The S&P 500 futures, expiring on December 18, is at 1,316.30. The appropriate number of futures is $20 million/ [(1,305)(250)] = 61.30.* **Sell 61 contracts**
December 18	The S&P 500 is at 1,300.36. The stocks will be worth (1,300.36/1305)($20 million) = $19.928889 million. The $20 million invested in the stocks effectively costs ($20 million)$e^{(0.052-0.03)(0.1096)}$ = $20.048242 million. Transactions costs are $20 million (0.005) = $100,000 (includes futures costs). **Sell stocks**	Futures expires at the S&P 500 price of 1,300.36. **Close out futures at expiration**

Analysis:

Profit on stocks:

$19,928,889 (received from sale of stocks)
−$20,048,282 (invested in stocks)
−$119,393.

Profit on futures:

61(250)(1,316.30) (sale price of futures)
−61(250)(1,300.36) (purchase price of futures)
$243,085.

Overall profit:

$243,085 (from futures)
−$119,393 (from stock)
−$100,000 (transaction costs)
$23,692.

*The appropriate number of futures contracts to match $20 million of stock is $20 million divided by the index price, *not the futures price*, times the multiplier. Even though the $20 million is allocated across 500 different stocks, it is equivalent to buying $20 million/1,305 = 15,326 "shares" of the S&P 500. The appropriate number of futures is one for each equivalent "share" of the S&P 500. Each futures, of course, has a $250 multiplier.

selling large quantities of stock and futures. Such actions have attracted considerable attention from the media. Critics have charged that program trading has led to increased volatility in the spot markets. Regulators and legislators have called for restrictions on such trading in the form of circuit breakers and reduced access to the DOT system for rapidly executing orders. Others have argued for imposing higher margins on futures trading. These issues continue to generate a lot of debate.

FOREIGN EXCHANGE ARBITRAGE

We have discussed carry arbitrage with Federal funds, Eurodollars, bond futures, and stock indices so far. The arbitrage concept is also applicable to foreign exchange futures. Recall that the model for the foreign exchange futures price is[7]

$$f_0(T) = S_0(1+r)^T/(1+\rho)^T,$$

where S_0 is the spot foreign exchange rate expressed in local currency per unit of foreign currency, r denotes the domestic risk free interest rate (annually compounded) and ρ denotes the foreign risk free interest rate (annually compounded). The continuously compounded equivalent is expressed as

$$f_0(T) = S_0 e^{(r_c - \rho_c)T},$$

where r_c is the continuously compounded risk-free rate and ρ_c is the continuously compounded foreign interest rate. Recall this relationship between the spot foreign exchange rate and the forward/futures price is known as interest rate parity.

Numerical Example Consider again (p. 277) the following example from a European perspective. On June 9 of a particular year, the spot rate for dollars was 0.7908 euros. The U.S. interest rate was 5.84 percent, while the euro interest rate was 3.59 percent. The time to expiration was 90/365 = 0.2466. Recall that we have

$$f_0(T) = €0.7908(1.0584)^{-0.2466}(1.0359)^{0.2466} = 0.7866 \text{ euros}.$$

Again, the forward rate should be about 0.7866 euros.

Now suppose that the observed market forward rate is 0.80 euros. Then, an arbitrage opportunity is available. An arbitrageur buys $(1.0584)^{-0.2466} = 0.9861$ dollars for \$0.9861(€0.7908) = 0.7798 euros and sells one forward contract at a forward rate of 0.80 euros. The 0.9861 dollars are invested at the U.S. risk-free rate. When the contract expires, the arbitrageur will have 1 dollar, which is delivered on the forward contract and for which 0.80 euros is received. Thus, the arbitrageur has invested 0.7798 euros and received 0.80 euros in 90 days. The annualized return is

$$\left(\frac{0.80}{0.7798}\right)^{365/90} - 1 = 0.1093,$$

which exceeds the euro risk-free rate of 3.59 percent. This transaction is called covered interest arbitrage. The combined effects of numerous arbitrageurs would push the spot rate up and/or the forward rate down until the spot and forward rates were properly aligned with the relative interest rates in the two countries.[8] Of course, some transaction costs and the dealer bid-ask spread would prevent the relationship from holding precisely.

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[7]In Chapter 9 we introduced foreign exchange forward contracts. We ignore any differences related to forwards and futures here.

[8]Arbitrage could also put pressure on interest rates in the two countries. The U.S. rate could decrease, while the euro rate could increase. Interest rates are, however, so heavily influenced by other effects, such as inflation, government borrowing, and central bank policy, that it is unlikely that arbitrageurs could influence rates.

DERIVATIVES TOOLS

Concepts, Applications, and Extensions
Currency-Hedged Cross-Border Index Arbitrage

We have now covered how to engage in covered interest arbitrage, in which a trader buys a currency and hedges its conversion back to the trader's currency using a forward or futures contract, and we have also covered how to engage in stock index arbitrage, in which the trader buys a portfolio of stock and sells a futures on an index that matches the portfolio. Now, we shall take a look at combining these two strategies.

Suppose that you are a Swiss equity trader who follows equities in the various European countries. The Dow Jones Euro STOXX 50 in particular is a euro-denominated index of 50 leading European stocks. A futures contract, also denominated in euros, trades on EUREX, the combined Swiss-German derivatives exchange. The trader observes that the futures seems to be overpriced. A stock index arbitrage transaction could be executed using an exchange-traded fund or ETF, which is a portfolio representing claims on a given index. To undertake this transaction, however, the trader would need to convert his own currency, Swiss francs, to euros; do the stock index arbitrage; and then convert the euros back to Swiss francs. Thus, the transaction is exposed to the risk of the euro-Swiss franc exchange rate. The trader knows, however, that he can hedge the conversion of euros back into Swiss francs, but the details are more complex than a straightforward conversion of euros to Swiss francs. The trader will have to take into account the value of the hedged portfolio when deciding on the size of the forward contract.

Suppose that the Dow Jones Euro STOXX index is at 2,664. A futures contract on the index is at 2680. Both numbers are in euros. The euro interest rate is 3 percent and the Swiss franc interest rate is 2.5 percent. These rates were based on LIBOR-type interest but have been converted to continuous compounding equivalents. The continuously compounded dividend yield on the index is 1.2 percent. The exchange rate is SF1.4726 per euro. The futures expires in exactly three months and each futures contract covers €10. That is, the quoted futures price is multiplied by 10 to obtain the actual futures price. ETFs on the index trade at 1/10 the price of the index. Thus, the ETF is at 266.40.

First, the trader calculates the theoretical price of the futures

$$2{,}664e^{(0.03-0.012)(0.25)} = 2{,}676.$$

Observing the futures price of 2,680, the trader knows that the futures is overpriced. He decides to sell 1,000 contracts. Accounting for the multiplier, this would be like selling futures to cover 1,000(10) = 10,000 shares. He must take into account, however, that if he purchases ETFs, the price is set at 1/10 of the index. Thus, he must purchase 100,000 ETF shares. In addition, if he purchases the shares, he will accrue dividends at the rate of 1.2 percent per year. He can reinvest these dividends into new shares. Since he wants the equivalent of 100,000 ETF shares after three months, he should purchase

$$100{,}000e^{-0.012(0.25)} = 99{,}700$$

shares of the ETF. He then sells 1,000 futures at 2,680. He will also need to engage in a forward contract to convert a specific amount of euros into Swiss francs. Since he will be effectively selling 10,000 shares of the index at 2,680, the amount of euros he should receive is 10,000(2,680) = 26,800,000. Thus, the forward contract should be written to cover €26,800,000. The forward price would be

$$1.4726e^{(0.025-0.03)(0.25)} = 1.4708.$$

The forward contract is assumed to be correctly priced, so the trader has entered into a commitment to deliver €26,800,000 at SF1.4708 per euro.

Thus, the trader buys the stock in the form of 99,700 ETF shares. This will require 99,700(€266.40) = €26,560,080, so he will have to commit €26,560,080 (SF1.4726/€) = SF39,112,374. Remember that the trader could earn 2.5 percent on this money by keeping it invested risk-free in Switzerland. He sells 1,000 futures at 2,680 and sells a forward contract on €26,800,000 at SF1.4708.

At expiration, the index is at S_T and the ETF is at $(1/10)S_T$. Due to the reinvestment of dividends, he now holds

$$99{,}700e^{0.012(0.25)} = 100{,}000$$

shares of the ETF, which are worth

$$100{,}000(1/10)S_T.$$

The futures payoff, ignoring the mark-to-market effect, is

$$-1{,}000(10)(S_T - 2{,}680).$$

Thus, the total payoff is

$$100{,}000(1/10)S_T - 1{,}000(10)(S_T - 2{,}680) = 26{,}800{,}000$$

euros. Using forward contracts, he converts this amount back into Swiss francs at the rate of SF1.4708 to obtain

$$€26{,}800{,}000(SF1.4708/€) = SF39{,}417{,}440.$$

Now let us see how well he has done. He invested SF39,112,374 and ended up with SF39,417,440. This is a return per Swiss franc invested of 39,417,440/39,112,374 = 1.00779973. The annualized continuously compounded rate of return can be found in the following manner:

$$\frac{39{,}417{,}440}{39{,}12{,}374} = e^{k(0.25)},$$

Then

$$\ln\left(\frac{39{,}417{,}440}{39{,}112{,}374}\right) = k(0.25),$$

and k will be 3.11 percent, which is earned risk-free. This is better than the 2.5 percent rate he could have gotten by investing risk-free in Switzerland. Of course, the trader must execute the trasactions quickly and cover all costs.

Questions and Problems

1. On November 1, the one-month LIBOR rate is 4.0 percent and the two-month LIBOR rate is 5.0 percent. Assume that Fed funds futures contracts trades at a 25 basis point rate under one-month LIBOR at the start of the delivery month. The December Fed funds futures is quoted at 94.75. Assuming no basis risk between Fed funds and one-month LIBOR at the start of the delivery month, identify whether an arbitrage opportunity is available. Contract size is $5,000,000. Be sure to illustrate the arbitrage strategy for one contract. To show the dollar arbitrage, assume the one-month LIBOR rate on December 1 was 7 percent.

2. Repeat problem 1, but now assume the one-month LIBOR rate on December 1 was 5.5 percent.
3. On July 5, a stock index futures contract was at 394.85. The index was at 392.54, the risk-free rate was 2.83 percent, the dividend yield was 2.08 percent, and the contract expired on September 20. Determine if an arbitrage opportunity was available, and explain what transactions were executed.
4. Rework problem 3 assuming that the index was at 388.14 at expiration. Determine the profit from the arbitrage trade, and express it in terms of the profit from the spot and futures sides of the transaction. How does your answer relate to that in problem 3?
5. It is August 20, and you are trying to determine which of two bonds is the cheaper bond to deliver on the December Treasury bond futures contract. The futures price is 89 12/32. Assume delivery will be made on December 14, and use 7.9 percent as the repo rate. Find the cheaper bond to deliver.
   a. Bond X: A 9 percent noncallable bond maturing in about 28 years with a price of 100 14/32 and a CF of 1.1106. Coupons are paid on November 15 and May 15. The accrued interest is 2.37 on August 20 and 0.72 on December 14.
   b. Bond Z: An 11 1/4 percent noncallable bond maturing in about 25 years with a price of 121 14/32 and a CF of 1.3444. Coupons are paid on February 15 and August 15. The accrued interest is 0.15 on August 20 and 3.7 on December 14.
6. On September 26 of a particular year, the March Treasury bond futures contract settlement price was 94-22. Compare the following two bonds and determine which is the cheaper bond to deliver. Assume delivery will be made on March 1. Use 5.3 percent as the repo rate.
   a. Bond A: A 12 3/4 percent bond callable in about 19 years and maturing in about 24 years with a price of 148 9/32 and a CF of 1.4433. Coupons are paid on November 15 and May 15. The accrued interest is 4.64 on September 26 and 3.73 on March 1.
   b. Bond B: A 13 7/8 percent bond callable in about 20 years and maturing in about 25 years with a price of 159 27/32 and a CF of 1.5689. Coupons are paid on November 15 and May 15. The accrued interest is 5.05 on September 26 and 4.06 on March 1.
7. Identify two ways to express interest rate parity based on how interest rates are quoted. Explain why, in practice, they contain the same information.
8. On March 16, the March T-bond futures settlement price was 101 21/32. Assume the 12 1/2 percent bond maturing in about 22 years is the cheapest bond to deliver. The CF is 1.4639. Assume that the price at 3:00 P.M. was 150 15/32. Determine the price at 5:00 P.M. that would be necessary to justify delivery.
9. On March 16, the June T-bond futures contract was priced at 100 17/32 and the September contract was at 99 17/32. Determine the implied repo rate on the spread. Assume the cheapest bond to deliver on both contracts is the 11 1/4 maturing in 28 years and currently priced at 140 21/32. The CF for delivery in June was 1.3593, and the CF for delivery in September was 1.3581. Delivery is on the first of the month, and the coupons are paid on February 15 and August 15. The accrued interest is 3.29 on June 1 and 6.16 on September 1.
10. On September 12, the cheapest-to-deliver bond on the December Treasury bond futures contract is the 9s of November 2018. The bond pays interest semiannually on May 15 and November 15. Its price is 125 12/32. The December futures price is 112 24/32. The bond has a conversion factor of 1.1002. Its accrued interest on September 12 is 2.91 and its accrued interest on December 1 is 4.92, which reflects the payment of the coupon on November 15. Assuming delivery on December 1, determine the implied repo rate. Then write an interpretation of your result.

11. (Concept Problem) In this chapter, there are two equations presented for the implied repo rate related to bond futures contracts shown below. Explain these equations and discuss the differences between them.

$$\hat{r}=\left[\frac{(CF)(f_0(T))+AI_t+CI_{0,T}}{B_0+AI_0}\right]^{(1/T)}-1 \text{ and}$$

$$\hat{r}=\left[\frac{(CF(t))\,f_0(t)+AI_t+CI_{t,T}}{(CF(t))f_0(t)+AI_t}\right]^{1/(T-t)}-1.$$

12. Assume that on March 16, the cheapest bond to deliver on the June T-bond futures contract is the 14s, callable in about 19 years and maturing in about 24 years. Coupons are paid on November 15 and May 15. The price of the bond is 161 23/32, and the CF is 1.584. The June futures price is 100 17/32. Assume a 5.5 percent reinvestment rate. Determine the implied repo rate on the contract. Interpret your result. Note that you will need to determine the accrued interest. Assume delivery on June 1.
13. Explain how the repurchase agreement plays a role in the pricing of futures contracts. What is the implied repo rate?
14. Explain the implied repo rate on a U.S. Treasury bond futures spread position.
15. Identify and discuss four non-traded delivery options related to U.S. Treasury bond futures contracts.
16. Define the conversion factor. Why are U.S. Treasury bond futures contracts designed with conversion factors?
17. Identify and explain some factors that make the execution of stock index futures arbitrage difficult in practice.
18. What is program trading? Why is it so controversial?
19. Explain the relationship between carry arbitrage and the implied repo rate.
20. On August 20 a stock index futures, which expires on September 20, was priced at 429.70. The index was at 428.51. The dividend yield was 2.7 percent. Discuss the concept of the implied repo rate on an index arbitrage trade. Determine the implied repo rate on this trade, and explain how you would evaluate it.
21. A corporate cash manager who often invests her firm's excess cash in the Eurodollar market is considering the possibility of investing $20 million for 180 days directly in a Eurodollar CD at 6.15 percent. As an alternative, she considers the fact that the 90-day rate is 6 percent and the price of a Eurodollar futures expiring in 90 days is 93.75 (the IMM index). She believes that the combination of the 90-day CD plus the futures contract would be a better way of lending $20 million for 180 days. Suppose she executes this strategy and the rate on 90-day Eurodollar CDs ninety days later is 5.9 percent. Determine the annualized rate of return she earns over 180 days and compare it to the annualized rate of return on the 180-day CD.
22. (Concept Problem) Referring to problem 3, suppose transaction costs amounted to 0.5 percent of the value of the stock index. Explain how these costs would affect the profitability and the incidence of index arbitrage. Then calculate the range of possible futures prices within which no arbitrage would take place.

Determining the CBOT Treasury Bond Conversion Factor

Step 1 Determine the maturity of the bond in years, months, and days as of the first day of the expiration month. If the bond is callable, use the first call date instead of the maturity date. Let YRS be the number of years and MOS the number of months. Ignore the number of days. Let c be the coupon rate on the bond.

Step 2 Round the number of months down to 0, 3, 6, or 9. Call this MOS*.

Step 3 If MOS* = 0,

$$CF_0 = \frac{c}{2}\left[\frac{1-(1.03)^{-2*YRS}}{0.03}\right] + (1.03)^{-2*YRS}$$

If MOS* = 3,

$$CF_3 = (CF_0 + c/2)(1.03)^{-0.5} - c/4.$$

If MOS* = 6,

$$CF_6 = \frac{c}{2}\left[\frac{1-(1.03)^{-(2*YRS+1)}}{0.03}\right] - (1.03)^{-(2*YRS+1)}$$

If MOS* = 9,

$$CF_9 = (CF_6 + c/2)(1.03)^{-0.5} - c/4.$$

Example: Determine the CF for delivery of the 5 1/4s of February 15, 2029, on the March 2006 T-bond futures contract.

On March 1, 2006, the bond's remaining life is 22 years, 11 months, and 14 days. Thus, YRS = 22 and MOS = 11. Rounding down gives MOS* = 9. First, we must find CF_6:

$$CF_6 = \frac{0.0525}{2}\left[\frac{1-(1.03)^{-(2(22)+1)}}{0.03}\right] + (1.03)^{-(2(22)+1)} = 0.908055,$$

Then we find CF_9 as

$$CF_9 = (1.908055 + 0.0525/2)(1.03)^{-0.5} - 0.0525/4 = 0.9075,$$

which is shown in Table 10.3 in the chapter.

The Excel spreadsheet CF7e.xls will automatically calculate the conversion factor for you. Software Demonstration 10.2 illustrates how to use CF7e.xls.

SOFTWARE DEMONSTRATION 10.2

Determining the CBOT Conversion Factor with the Excel Spreadsheet CF7e.xls

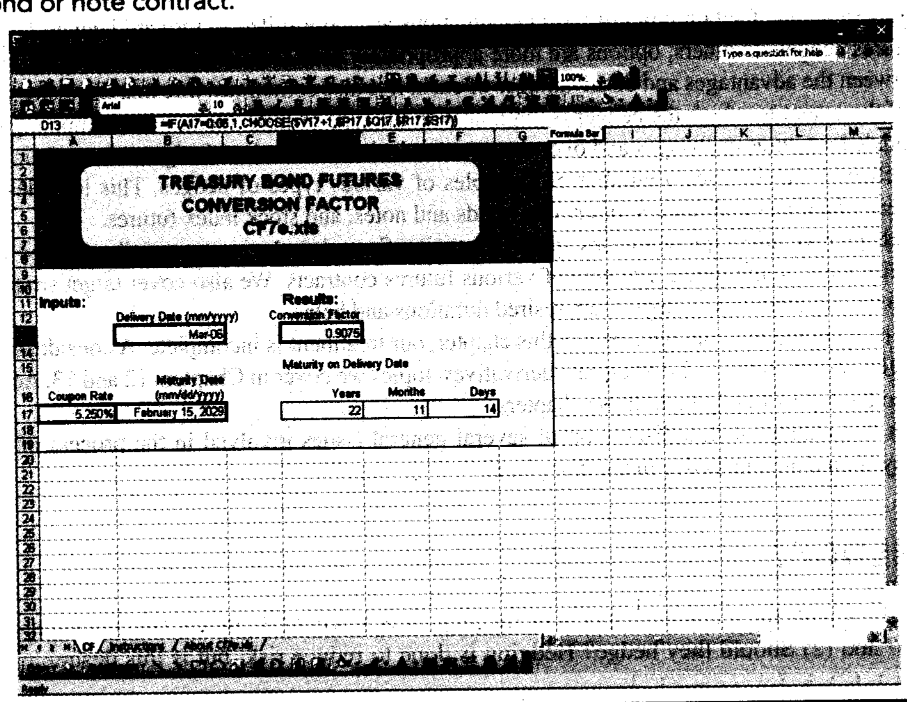
The Excel spreadsheet CF7e.xls is written in Excel 2002. It calculates the conversion factor for a bond delivered on the Chicago Board of Trade's Treasury bond futures contract. To use the spreadsheet, you will need Windows 95 or higher. The spreadsheet is available as a download via the product support Web site. To access it:

1. Go to www.academic.cengage.com/aise.
2. Click on Instructor Resources or Student Resources.
3. Locate the title of this book on the Web site.
4. Download and install the spreadsheet using the link provided.

This spreadsheet is a read-only spreadsheet, meaning that it cannot be saved under the name CF7e.xls, which preserves the original file.

Now let us work an example. Suppose we wish to deliver the 5 1/4 of February 15, 2029 on the March 2006 contract. Each cell that will accept input has a double-line border and the values are in blue. In the section labeled **Inputs:** you should enter the delivery date. This will always be simply a month and a year. Enter it in the form mm/yyyy. Thus, you should insert "3/2006" in the cell. Several rows below, you should enter the coupon rate and maturity date. Enter the coupon rate as a percentage or decimal. For example, our bond's coupon is 5.25, so enter "5.25" or "0.0525". Enter the maturity date in the format mm/dd/yyyy. Thus, in this case, you would enter "2/15/2029". The spreadsheet displays both of these dates in Excel's date format. Press F9 (manual recalculation).

The results are then calculated and appear in the section labeled **Results:** Output values have a single-line border. The conversion factor here is 0.9075. The spreadsheet also shows the maturity in years (here, 22), months (11), and days, based on the first day of the expiration month, (14). This enables you to see if the bond has a sufficiently long maturity to be eligible for delivery on the bond or note contract.



11

FORWARD AND FUTURES HEDGING, SPREAD, AND TARGET STRATEGIES

Hedging is a type of transaction designed to reduce or, in some cases, eliminate risk. Our material on options presented numerous examples of hedges, the most obvious being the covered call and protective put. Now we shall find that it is also possible, in some cases preferable, to use forwards or futures to hedge.

Until now we have emphasized that there are many similarities and differences between forward and futures contracts. Both can be used for hedging. When choosing a forward or futures hedge over an option hedge, the hedger agrees to give up future gains and losses. No up-front cost is incurred. In contrast, an option hedge such as a protective put preserves future gains but at the expense of an up-front cost, the option premium. Forward contracting, as we previously noted, involves some credit risk and is generally available only in very large transaction sizes. Forward contracting, however, allows the user to customize the terms of the transaction so as to get a near-perfect hedge. Certain types of business situations are more suited to forward hedges. Others are more suited to futures hedges. In others, options are more appropriately used. While the choice of instrument is often a trade-off between the advantages and disadvantages of each, it is also sometimes true that the instrument chosen is a function of the extent to which the hedger is familiar with the type of instrument, what competing firms do, and the efforts made by futures exchanges and over-the-counter dealers to convince hedgers to use their products.

In this chapter we shall see a number of examples of various types of hedges. This includes hedges with foreign currency forwards and futures, futures on bonds and notes, and stock index futures.

In this chapter we also explore various spread strategies. Spread strategies are usually pursued when a trader has a particular view on the future direction of various futures contracts. We also cover target strategies wherein investors use futures contracts to target their desired durations and betas.

In spite of all of the material we cover in this chapter, our treatment is incomplete. A considerable number of hedge strategies utilize swaps and interest rate derivatives, topics we cover in Chapters 12 and 13. Accordingly, we shall return to the topic of hedging in those chapters.

We start this chapter with a discussion of several general issues involved in the process of hedging. For example, why should anyone hedge in the first place?

WHY HEDGE?

Before we begin with the technical aspects of hedging, it is worthwhile to ask two questions: (1) Why do firms hedge? and (2) Should they hedge? Hedging is done to reduce risk, but is this desirable? If everyone hedged, would we not simply end up with an economy in which no one takes risks? This would surely lead to economic stagnancy. Moreover, we must wonder whether hedging can actually increase shareholder wealth.[1]

If the famous Modigliani-Miller propositions are correct, then the value of the firm is independent of any financial decisions, which include hedging. Hedging, however, may be desired by the shareholders simply to find a more acceptable combination of expected return and risk. It can be argued, however, that firms need not hedge since shareholders, if they wanted hedging, could do it themselves. But this ignores several important points. It assumes that shareholders can correctly assess all the firm's risks that can be hedged. If a company is exposed to the risk associated with volatile raw materials prices, can the shareholders properly determine the degree of risk? Can they determine the periods over which that risk is greatest? Can they determine the correct number of futures contracts necessary to hedge their share of the total risk? Do they even qualify to open a futures brokerage account? Will their transaction costs be equal to or less than their proportional share of the transaction costs incurred if the firm did the hedging? The answer to each of these questions is "maybe not." It should be obvious that hedging is not something that shareholders can always do as effectively as firms.

Corporate hedging activities may be related to the relative cost of internal and external financing. Froot, Scharfstein, and Stein suggest that corporate hedging is motivated by a desire to ensure lower cost financing internally when attractive investment opportunities are available.

In addition, there may be other reasons why firms hedge, such as tax advantages. Low-income firms, for example those that are below the highest corporate tax rate, can particularly benefit from the interaction between hedging and the progressive corporate income tax structure.[2] Hedging also reduces the probability of bankruptcy. This is not necessarily valuable to the shareholders except that it can reduce the expected costs that are incurred if the firm does go bankrupt.

A firm may choose to hedge because its managers' livelihoods may be heavily tied to the performance of the firm. The managers may then benefit from reducing the firm's risk. This may not be in the shareholders' best interests, but it can at least explain why some firms hedge. Finally, hedging may send a signal to potential creditors that the firm is making a concerted effort to protect the value of the underlying assets. This can result in more favorable credit terms and less costly, restrictive covenants.

Many firms, such as financial institutions, are constantly trading over-the-counter financial products like swaps and forwards on behalf of their clients. They offer these services to help their clients manage their risks. These financial institutions then turn around and hedge the risk they have assumed on behalf of their clients. How do they make money? They quote rates and prices to their clients that reflect a spread sufficient to cover their hedging costs and include a profit. In this manner, they become retailers of hedging services.

Lest we give a one-sided view of hedging, it is important to consider some reasons not to hedge. One reason is that hedging can give a misleading impression of the amount of risk reduced. There is an old saying in derivatives: "The only perfect hedge is in a Japanese garden." Hedges nearly always leave some risk and some hedges leave a surprising amount of risk that was supposed to have been eliminated. Hedging should always be viewed as risk reducing but not eliminating, thus requiring that the remaining risk be identified and monitored.

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[1]The material in this section draws heavily from C. W. Smith and R. M. Stulz, "The Determinants of Firms' Hedging Policies," *Journal of Financial and Quantitative Analysis* 20 (1985): 391–405; D. Duffie, "Corporate Risk Management 101: Why Hedge?" *Corporate Risk Management* 3 (May 1991): 22–25; D. R. Nance, C. W. Smith Jr., and C. W. Smithson, "On the Determinants of Corporate Hedging," *The Journal of Finance* 48 (March 1993): 267–284; and K. A. Froot, D. S. Scharfstein, and J. C. Stein, "Risk Management: Coordinating Corporate Investment and Financing Policies," *The Journal of Finance* 48 (December 1993): 1629–1658.

[2]See Appendix 11 for more on hedging and taxes.

Another problem with hedging is that it eliminates the opportunity to take advantage of favorable market conditions. In other words, hedging reduces the gain potential as well as the loss potential. Carried to an extreme, hedging can nearly eliminate any reason for being in business in the first place. The creation of wealth does not come about by indiscriminate hedging. Hedging should be selective—that is, reducing certain risks while maintaining exposures where an advantage is perceived.

Finally, we should add that *there is really no such thing as a hedge.* As surprising as that sounds, consider this. When an investor moves all funds from stock to cash, he may think he is hedging, but he is really taking a position that the market will go down. When a corporation hedges away the risk associated with borrowing at a floating rate, it is taking a position that interest rates will go up. In either case, the elimination of risk is taking a position based on a view that an unfavorable event will occur in the market. This is as much of a speculative action as is investing all of one's money in the stock market or borrowing at a floating rate. In other words, reducing risk is a bet that bad things will happen in the market.

With all of this talk about hedging, however, we would be remiss not to note that hedging is just a part of an overall process called *risk management.* Hedging is a specific example of managing risk for the purpose of reducing it. In a broader sense, however, there is much more to managing risk than just hedging. In some situations, risk will be lower than desired, calling for an increase in risk. Is this the opposite of hedging? Some would indeed call it *speculation,* but in fact, it is just part of the overall process of risk management, the alignment of the actual level of risk with the desired level of risk. While the focus of this chapter is on hedging, we shall discuss the bigger picture of managing risk in Chapters 15 and 16.

HEDGING CONCEPTS

Before we can understand why a certain hedge is placed or how it works, we must become acquainted with a few basic hedging concepts. We have mentioned some of these points before but have not specifically applied them to hedging strategies. In the discussion below, we shall primarily refer to futures, but the ideas are nearly always applicable to forward contracts as well.

Short Hedge and Long Hedge

The terms short hedge and long hedge distinguish hedges that involve short and long positions in the futures contract, respectively. A hedger who holds an asset and is concerned about a decrease in its price, such as a grain elevator operator owning a large inventory of wheat, might consider hedging it with a short position in futures. If the spot price and futures price move together, the hedge will reduce some of the risk. For example, if the spot price decreases, the futures price also will decrease. Since the hedger is short the futures contract, the futures transaction produces a profit that at least partially offsets the loss on the spot position. This is called a *short hedge* because the hedger is short futures. The grain elevator operator is able to hedge price risk with a short wheat futures position.

Another type of hedge situation is faced when a party plans to purchase an asset at a later date, such as a cereal producer. Fearing an increase in wheat prices, the cereal producer might buy futures contracts. Then, if the price of wheat increases, the wheat futures price also will increase and produce a profit on the futures position. That profit will at least partially offset the higher cost of purchasing wheat. This is a *long hedge,* because the hedger, the cereal producer in this example, is long in the futures market. Because it involves an anticipated transaction, it is sometimes called an anticipatory hedge.

Another type of long hedge might be placed when one is short an asset. Although this hedge is less common, it would be appropriate for someone who has sold short a stock and is concerned that the market will go up. Rather than close out the short position, one might buy a futures contract and earn a profit on the long position in futures that will at least partially offset the loss on the short position in the stock.

Table 11.1 Summary of Hedging Situations

Condition Today	Risk	Appropriate Hedge
Hold asset	Asset price may fall	Short hedge
Plan to buy asset	Asset price may rise	Long hedge
Sold short asset	Asset price may rise	Long hedge

Note: Short hedge means long spot, short futures; long hedge means short spot, long futures. Hedging situations involving loans are examined in Chapters 12 and 13.

In each of these cases, the hedger held a position in the spot market that was subject to risk. The futures transaction served as a temporary substitute for a spot transaction. Thus, when one holds the asset and is concerned about a price decrease but does not want to sell it, one can execute a short futures trade. Selling the futures contract would substitute for selling the commodity. Table 11.1 summarizes these various hedging situations.

The Basis

The basis is one of the most important concepts in futures markets because it aids in understanding the process of hedging. The basis usually is defined as the spot price minus the futures price. Some books and articles, however, define it as the futures price minus the spot price. In this book, we shall use the former definition:

$$\text{Basis} = \text{Spot price} - \text{Futures price.}$$

Hedging and the Basis Here we will look at the concept of hedging and how the basis affects the performance of a hedge. Ultimately we shall need to understand the factors that influence the basis.

Let us define the following terms:

T = time point of expiration (say a particular month, day, and year)

t = time point prior to expiration ($t = 0$ implies "today")

S_0 = spot price today

f_0 = futures price today

S_T = spot price at expiration

f_T = futures price at expiration

S_t = spot price at time t prior to expiration

f_t = futures price at time t prior to expiration

Π = profit from a given strategy

For the time being, we shall ignore marking to market, any costs of storing the asset, and other transaction costs.

The concept of a hedge is not new. When we looked at options, we constructed several types of hedges, some of which were riskless. By taking a position in a stock and an opposite position in an option, gains (losses) on the stock are offset by losses (gains) on the option. We can do the same thing with futures: hold a long (short) position in the spot market and a short (long) position in the futures market. For a long position in the spot market, the profit from a hedge held to expiration is

$$\Pi \text{ (short hedge)} = (S_T - S_0) \text{ (spot market profit)} + (f_0 - f_T) \text{ (futures market profit).}$$

Recall that profit is always the selling price minus the purchase price. The futures market profit from a short position is $(f_0 - f_T)$ or $-(f_T - f_0)$. For a short position in the spot market and a long position in the futures market, the sign of each term in the above equation is reversed; that is,

$$\Pi \text{ (long hedge)} = (S_0 - S_T) \text{ (spot market profit)} + (f_T - f_0) \text{ (futures market profit)}.$$

In some cases, we might wish to close out the position at time t, that is, before expiration. Then the profits from a short hedge and a long hedge are

$$\Pi \text{ (short hedge)} = (S_t - S_0) + (f_0 - f_t),$$
$$\Pi \text{ (long hedge)} = (S_0 - S_t) + (f_t - f_0).$$

At expiration, a person buying a futures contract can expect to receive immediate delivery of the good. Thus, an expiring futures contract is the same as the purchase of the spot commodity; therefore, $S_T = f_T$. Thus, the profit if the short hedge is held to expiration is simply $f_0 - S_0$. That means that the hedge is equivalent to buying the asset at price S_0 and immediately guaranteeing a sale price of f_0. Also, the profit if the long hedge is held to expiration is $S_0 - f_0$, which is equivalent to selling the asset at price S_0 and immediately guaranteeing the purchase price of f_0.

As an example, suppose you buy an asset for $100 and sell a futures contract on the asset at $103. Therefore, you have a short hedge. At expiration, the spot and futures prices are both $96. You sell the asset for $96, taking a $4 loss, and close your futures contract at $96, making a $7 gain, for a net profit of $3. Alternatively, you could deliver the asset on your futures contract, receiving $96, and collect the $7 that has accumulated in your futures account, making the effective sale price of the asset $103. In either case, the transaction is equivalent to selling the asset for $103, the original futures price.

Now suppose instead you short sell an asset for $100 and buy a futures contract on the asset at $103. Therefore, you have a long hedge. At expiration, the spot and futures prices are both $96. You buy back the asset for $96, receiving a $4 gain, and close your futures contract at $96, taking a $7 loss, for a net loss of $3. Alternatively, you could purchase the asset on your futures contract, paying $96 and with the $7 loss that has accumulated in your futures account, the effective purchase price of the asset is $103. In either case, the transaction is equivalent to purchasing the asset for $103, the original futures price.

Since the basis is defined as the spot price minus the futures price, we can write it as a variable, b, where

$$b_0 = S_0 - f_0 \text{ (initial basis)}$$
$$b_t = S_t - f_t \text{ (basis at time t)}$$
$$b_T = S_T - f_T \text{ (basis at expiration).}$$

Thus, for positions closed out at time t,

$$\Pi \text{ (short hedge)} = (S_t - f_t) - (S_0 - f_0),$$
$$\Pi \text{ (short hedge)} = b_t - b_0$$
$$\Pi \text{ (long hedge)} = (S_0 - f_0) - (S_t - f_t),$$
$$\Pi \text{ (long hedge)} = b_0 - b_t$$

The profits from the hedges are simply the change in the basis. The uncertainty regarding how the basis will change is called basis risk. A hedge substitutes the change in the basis for the change in the spot price. The basis change usually is far less variable than the spot price change; hence, the hedged position is less risky than the unhedged position. Because basis risk results from the uncertainty over the change in the basis, hedging is a speculative activity but produces a risk level much lower than that of an unhedged position.

Hedging can also be viewed as a transaction that attempts to establish the expected future price of an asset. A short (long) hedge establishes the expected future sales (purchase) price. For example, the equation for the profit on a short hedge can be written as $f_0 + (S_t - f_t) - S_0$. Since the short hedger paid S_0 dollars to purchase the asset, then the effective sale price of the asset can be viewed as $f_0 + (S_t - f_t)$, which can be written as $f_0 + b_t$. In other words, a short hedge establishes the future sale price of the asset as the current futures price plus the basis. Since f_0 is known, the effective future sale price is uncertain only to the extent that the basis is uncertain. If the short hedge is held to expiration, the effective sale price becomes $f_0 + b_T$, which is simply f_0 since the basis at expiration is zero. Thus, the sale price of the asset is established as f_0 when the transaction is initiated. The short hedger would be sure that she would be able to effectively sell the asset for the futures price. This would be a 'perfect' short hedge. Most hedges are imperfect for a variety of reasons but often because the hedger does not typically hold the position to expiration.

If the spot price increases by more than the futures price, the basis will increase. This is said to be a strengthening basis, and it improves (reduces) the performance of the short (long) hedge. If the futures price increases by more than the spot price, the basis will decrease, reducing (improving) the performance on the short (long) hedge. In that case, the basis is said to be weakening. These relationships between hedging profitability and the basis are summarized in Table 11.2.

As noted above, if a hedge is held all the way to expiration, the basis goes to zero. In that case, the profit is simply $-b_0$ from a short hedge and $+b_0$ from a long hedge.

Finally, we should remember that hedging incurs costs, such as the transaction costs of the futures. In addition, if the asset is held, it will incur costs of storage. These will reduce the profit but their effects are generally known in advance and, thus, do not impose any additional risk.

Hedging Example Let us consider an example using gold. On March 30, the price of a gold futures expiring in June was \$388.60 per troy ounce. The spot price of gold was \$387.15. Suppose a gold dealer held 100 troy

Table 11.2 Hedging Profitability and the Basis

Type of Hedge	Benefits from	Which Occurs if
Short hedge	Strengthening basis	Spot price rises more than futures price rises or Spot price falls less than futures price falls or Spot price rises and futures price falls
Long hedge	Weakening basis	Spot price rises less than futures price rises or Spot price falls more than futures price falls or Spot price falls and futures price rises

Note: Short hedge means long spot, short futures; long hedge means short spot, long futures.

ounces of gold worth 100(387.15) = 38,715. We shall disregard the storage costs, because they are reasonably certain. To protect against a decrease in the price of gold, the dealer might sell one futures contract on 100 troy ounces, hence he has entered a short hedge. In our notation,

$$S_0 = 387.15$$

$$f_0 = 388.60$$

$$b_0 = 387.15 - 388.60 = -1.45.$$

If the hedge is held to expiration, the basis should converge to zero. It might not go precisely to zero, however, and we shall see why later. If it does, the profit should be −1 times the original basis times the number of ounces:

$$\Pi = -1(-1.45)(100) = 145.$$

Suppose that at expiration the spot price of gold is \$408.50. Then the dealer sells the gold in the spot market for a profit of 100(408.50 − 387.15) = 2,135. The short futures contract is offset by purchasing it in the futures market for a profit of −100(408.50 − 388.60) = −1,990. The overall profit therefore is −1,990 + 2,135 = 145, as we predicted.

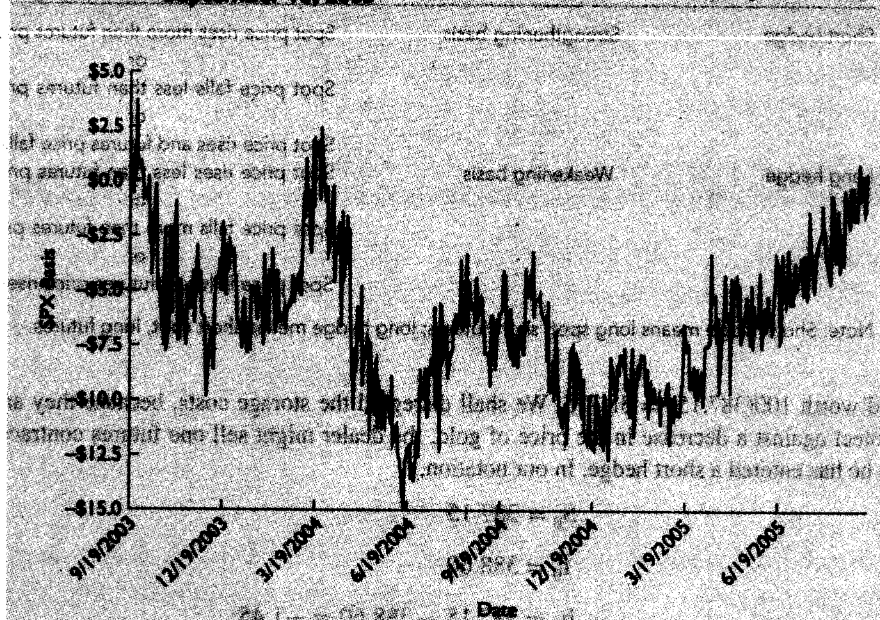
Now suppose we close the position prior to expiration. For example, on May 5 the spot price of gold was \$377.52 and the June futures price was \$378.63. In our notation, $S_t = 377.52$ and $f_t = 378.63$. If the gold is sold in the spot market, the profit is 100(377.52 − 387.15) = −963. The futures contract is bought back at 378.63 for a profit of −100(378.63 − 388.60) = 997. The net gain is −963 + 997 = 34. As we said earlier, this should equal the change in the basis, $b_t - b_0$. The original basis was −1.45. The basis when the position is closed is $S_t - f_t$, or 377.52 − 378.63 = −1.11. The profit, therefore, is

$$-1.11 - (-1.45) = 0.34,$$

which is the gain on the hedge per ounce of gold.

Behavior of the Basis Figure 11.1 shows the basis on a September S&P 500 index futures contract for a two-year period prior to expiration. Notice that the basis is quite volatile but finally converges to the spot price as maturity approaches. The volatility of the basis can be attributed to changing interest rates, changing expectations about dividend payments and asynchronous prices. Asynchronous pricing refers to pricing being set at different points in time. The S&P 500 index closing value is set based on the closing prices of the 500 stocks in the index. The stock markets close at 3:00 PM EST. The S&P 500 index futures markets close at 3:15 PM EST, hence there is a slight timing difference.

FIGURE 11.1 The September S&P 500 Index Basis, September 19, 2003–September 15, 2005



The basis does not always converge exactly to zero. In the case of gold, for example, an investor who purchased gold on the spot market and immediately sold a futures contract that is about to expire would have to deliver the gold. There is a potentially significant delivery cost that could leave the futures price slightly above the spot price. For some commodities, there are several acceptable grades that can be delivered, so there are multiple spot prices. The short has control over the delivery and will choose to deliver the most economical grade. The futures price will tend to converge to the spot price of the commodity that is most likely to be delivered.

Of course, in forward markets the basis is rarely an issue. The hedge is customized so the basis risk can be eliminated.

Some Risks of Hedging

Sometimes the asset being hedged and the asset underlying the futures contract differ. A typical example, which we shall illustrate later, is the hedging of a corporate bond with a Treasury bond futures contract. This is referred to as a cross hedge and is a type of basis risk much greater than that encountered by hedging government bonds with Treasury bond futures. Corporate and government bond prices tend to move together, but the relationship is weaker than that of two government bonds. In addition, bonds with higher ratings would be more highly correlated with government bonds. Thus, lower-quality corporate bonds would carry some additional basis risk, and hedges would tend to be less effective.

In some cases, the price of the asset being hedged and that of the futures contract move in opposite directions. Then a hedge will produce either a profit or a loss on both the spot and the futures positions. If one chooses the correct futures contract, this is unlikely to occur. If it occurs frequently, the hedger should find a different contract.

Hedging also entails another form of risk called quantity risk. For instance, suppose a farmer wishes to lock in the selling price of the crop that has not yet been harvested. The farmer might sell a futures contract and thereby establish the future selling price of the crop. Yet what the farmer does not know and cannot hedge is the uncertainty over the size of the crop. This is the farmer's quantity risk. The farmer's total revenue is the product of the crop's price and its size. In a highly competitive market, the farmer's crop is too small to influence the price, but there are systematic factors, such as weather, that could influence everyone's crop. Thus, the crop size could be small when prices are high and large when prices are low. This situation creates its own natural hedge. When the farmer hedges, the price volatility no longer offsets the uncertainty of the crop size. Thus, the hedge actually can *increase* the overall risk. Quantity uncertainty is common in farming but is by no means restricted to it. Many corporations and financial institutions do not know the sizes of future cash flows and thus must contend with quantity risk.

In the ideal hedge, the so-called perfect hedge, the hedger knows the horizon date on which he will enter into the spot transaction that he is trying to hedge. He would use a futures or another derivative that expires on that exact date. In some cases, however, the hedger does not know the horizon date for the hedge. For example, the owner of an asset might know for sure that he will need to sell the asset at a future date, but he might not know the exact date on which the sale will take place. In another situation, a party might know that she will receive some cash at a future date and will use the cash to purchase a particular asset. But she may not know the exact date on which the cash will be received. When the hedger does not know the horizon date, it will be more difficult to align the expiration date of the futures or forward contract with the hedge horizon date. Then the effectiveness of the hedge will be lower.

Contract Choice

When using futures to hedge, the choice of contract actually consists of four decisions: (1) which futures underlying asset, (2) which expiration month, (3) whether to be long or short, and (4) the number of contracts. The number of contracts is so important that we defer it to the next main section.

Which Futures Underlying Asset? From the previous section, we can see that it is important to select a futures contract on an asset that is highly correlated with the underlying asset being hedged. In many cases the choice is obvious, but in some it is not.

For example, suppose that a party wishes to hedge the value of a highly diversified portfolio of mid-cap stocks. There is a futures contract on the S&P MidCap Index, a measure of the performance of 400 medium-sized stocks. But the portfolio at risk does not perfectly match the S&P MidCap Index. Moreover, the futures contract on this index is not all that actively traded. Thus, it is possible that the hedger would want to use a futures contract on a more actively traded large- or small-cap index. Of course, if the hedger wanted a perfect match between the portfolio and the index underlying the hedging contract, he would choose a forward contract customized to match his portfolio, but the higher costs might be a factor in choosing the standardized futures contract.

Another factor one should consider is whether the contract is correctly priced. A short hedger will be selling futures contracts and therefore should look for contracts that are overpriced or, in the worst case, correctly priced. A long hedger should hedge by buying underpriced contracts or, in the worst case, correctly priced contracts.

Sometimes the best hedge can be obtained by using more than one futures commodity. For example, a hedge of a jet fuel position might be more effective if both heating oil futures *and* crude oil futures are used.

Which Expiration? Once one has selected the futures commodity, one must decide on the expiration month. As we know, only certain expiration months trade at a given time. For example, in September the Treasury bond futures contract has expirations of December of the current year, March, June, September, and December of the following year, and March, June, and September of the year after that. If the Treasury bond futures contract is the appropriate hedging vehicle, the contract used must come from this group of expirations.

In most cases, the hedger knows the time horizon over which the hedge must remain in effect. To obtain the maximum reduction in basis risk, a hedger should hold the futures position until as close as possible to that date. Thus, an appropriate contract expiration would be one that corresponded as closely as possible to the horizon date. The general rule of thumb, however, is to avoid holding a futures position in the expiration month. This is because unusual price movements sometimes are observed in the expiration month, and this would pose an additional risk to hedgers. Thus, the hedger should choose an expiration month that is as close as possible to but after the month in which the hedge is terminated.[3]

Table 11.3 lists possible hedge termination dates for a Treasury bond futures hedge and the appropriate contracts for use. Consider, however, that *the longer the time to expiration, the less liquid the contract.*

Table 11.3 Contract Expirations for Planned Hedge Termination Dates (Treasury Bond Futures Hedge Initiated on September 30, 2006)

Hedge Termination Date	Appropriate Contract
10/1/06–11/30/06	Dec 06
12/1/06–2/28/07	Mar 07
3/1/07–5/31/07	Jun 07
6/1/07–8/31/07	Sep 07
9/1/07–11/30/07	Dec 07
12/1/07–2/28/08	Mar 08
3/1/08–5/31/08	Jun 08
6/1/08–8/31/08	Sep 08
9/1/08–11/30/08	Dec 08

Note: The appropriate contract is based on the rule that the expiration date should be as soon as possible after the hedge termination date, subject to no contract being held in its expiration month. Liquidity considerations may make some more distant contracts inappropriate.

[3]Not all contracts exhibit unusual price behavior in the expiration month. Thus, this rule need not always be strictly followed.

Therefore, the selection of a contract according to this criterion may need to be overruled by the necessity of using a liquid contract. If this happens, one should use a contract with a shorter expiration. When the contract moves into its expiration month, the futures position is closed out and a new position is opened in the next expiration month. This process, called *rolling the hedge forward,* generates some additional risk but can still be quite effective.

Of course, the time horizon problem can be handled perfectly by using a forward contract from the over-the-counter market. In fact, some hedgers have horizons of longer than ten years, which can be hedged only by using forward contracts or swaps.

Long or Short? Regardless of whether one uses forwards or futures, the most important decision is whether to be long or short. There is absolutely no room for a mistake here. If a hedger goes long (or short) when she should have been short (or long), she has doubled the risk. The end result will be a gain or loss twice the amount of the gain or loss of the unhedged position.

The decision of whether to go long or short requires a determination of which type of market move will result in a loss in the spot market. It then requires establishing a forward or futures position that will be profitable while the spot position is losing. Table 11.4 summarizes three methods that will correctly identify the appropriate transaction. The first method requires that the hedger identify the worst case scenario and then establish a forward/futures position that will profit if the worst case does occur. The second method requires taking a forward/futures position that is opposite to the current spot position. This is a simple method, but in some cases it is difficult to identify the current spot position. The third method identifies the spot transaction that will be conducted when the hedge is terminated. The forward/futures transaction that will be conducted when the hedge is terminated should be the opposite of this spot transaction. The forward/futures transaction that should be done today should be the opposite of the forward/futures transaction that should be done at the termination of the hedge.

Table 11.4 How to Determine Whether to Buy or Sell Forwards/Futures When Hedging

Worst Case Scenario Method

1. Assuming that the spot and forward/futures markets move together, determine whether long and short positions in forward/futures would be profitable if the market goes up or down.
2. What is the worst that could happen in the spot market?
   a. The spot market goes up.
   b. The spot market goes down.
3. Given your answer in 2, assume that the worst that *can* happen *will* happen.
4. Given your answer in 3, and using your answer in 1, take a forward/futures position that will be profitable.

Current Spot Position Method

1. Determine whether your current position in the spot market is long or short.
   a. If you own an asset, your current position is long.
   b. If you are short an asset, your current position is short.
   c. If you are committed to buying an asset in the future, your current position is short.
2. Take a forward/futures position that is opposite the position given by your answer in 1.

Anticipated Future Spot Transaction Method

1. Determine what type of spot transaction you will be making when the hedge is terminated.
   a. Sell an asset.
   b. Buy an asset.
2. Given your answer in 1, you will need to terminate a forward/futures position at the horizon date by doing the opposite transaction to the one in 1, e.g., if your answer in 1 is "sell," your answer here is "buy a forward/futures."
3. Given your answer in 2, you will need to open a forward/futures contract today by doing the opposite, e.g., if your answer in 2 is "buy a forward/futures," your answer here should be "sell a forward/futures."

Margin Requirements and Marking to Market

Two other considerations in hedging with futures contracts are the margin requirement and the effect of marking to market. We discussed these factors earlier, but now we need to consider their implications for hedging.

Margin requirements, as we know, are very small and virtually insignificant in relation to the size of the position being hedged. Moreover, margin requirements for hedges are even smaller than speculative margins. In addition, margins can sometimes be posted with risk-free securities; thus, interest on the money can still be earned. Therefore, the initial amount of margin posted is really not a major factor in hedging.

What is important, however, is the effect of marking to market and the potential for margin calls. Remember that the profit on a futures transaction is supposed to offset the loss on the spot asset. At least part of the time, there will be profits on the spot asset and losses on the futures contract. On a given day when the futures contract generates a loss, the hedger must deposit additional margin money to cover that loss. Even if the spot position has generated a profit in excess of the loss on the futures contract, it may be impossible, or at least inconvenient, to withdraw the profit on the spot position to cover the loss on the futures.

This is one of the major obstacles to more widespread use of futures. Because futures profits and losses are realized immediately and spot profits and losses do not occur until the hedge is terminated, many potential hedgers tend to weigh the losses on the futures position more heavily than the gains on the spot. They also tend to think of hedges on an ex post rather than ex ante basis. If the hedge produced a profit on the spot position and a loss on the futures position, it would be apparent after the fact that the hedge was not the best that could have been done. But this would not be known *before the fact*.

Thus, a hedger must be aware that hedging will produce both gains and losses on futures transactions and will require periodic margin calls. The alternative to not meeting a margin call is closing the futures position. It is tempting to do this after a streak of losses and margin calls. If the futures position is closed, however, the hedge will no longer be in effect and the individual or firm will be exposed to the risk in the spot market, which is greater than the risk of the hedge.

In Chapter 9, we examined the effect of marking to market on the futures price. We concluded that the impact is fairly small. If, however, the interest earned or paid on the variation margin is not insignificant, it is possible to take it into account when establishing the optimal number of contracts. We shall cover this topic in a later section.

Of course, forward contracts do not entail margin requirements and marking to market, but they are subject to credit risk. Indeed, margin requirements and marking to market are primarily used by futures markets to reduce, if not effectively eliminate, credit risk.

These are several of the most important factors one must consider before initiating a hedge. As we noted earlier, another important consideration is the size of the hedge transaction. With forward hedges, it is a fairly simple matter to determine the appropriate size. For example, if a position of $10 million in an asset with a horizon date in exactly 60 days is to be hedged with a forward contract, the hedger simply specifies that he wants a forward contract covering $10 million of an asset with an expiration in 60 days. For futures contracts, the decision is not that simple. First, as noted, the asset underlying the futures may not match the asset being hedged. Second, even if the asset underlying the futures contract does match the asset being hedged, futures contracts may not be available in denominations that, in multiples, would equal $10 million. For example, the standard size could be $3 million. Third, the hedge horizon date may not correspond to the expiration date of any of the available futures contracts. Assuming that the hedger has chosen the underlying contract, he will need to carefully decide how many futures contracts to use to balance the risk being added by the futures contract with the risk that he is trying to hedge. The appropriate number of futures contracts is called the hedge ratio.

DETERMINATION OF THE HEDGE RATIO

The hedge ratio is the number of futures contracts one should use to hedge a particular exposure in the spot market.[4] The hedge ratio should be the one in which the futures profit or loss matches the spot profit or loss. There is no exact method of determining the hedge ratio before performing the hedge. There are, however, several ways to estimate it.

The most elementary method is to take a position in the futures market equivalent in size to the position in the spot market. For example, if you hold $10 million of the asset, you should hold futures contracts covering $10 million. If each futures contract has a price of $80,000, you should sell 125,000 contracts. This approach is relatively naive, because it fails to consider that the futures and spot prices might not change in the same proportions. In some cases, however, particularly when the asset being hedged is the same as the asset underlying the futures contract, such a hedge ratio will be appropriate.

Nevertheless, in most other cases the futures and spot prices will change by different percentages. Suppose we write the profit from a hedge as follows:

$$\Pi = \Delta S + \Delta f N_f,$$

where the symbol Δ means *change in.* Thus, the profit is the change in the spot price (ΔS) plus the change in the futures price (Δf) multiplied by the number of futures contracts (N_f). A positive N_f means a long position and a negative N_f means a short position. For the futures profit or loss to completely offset the spot loss or profit, we set $\Pi = 0$ and find the value of N_f as

$$N_f = -\frac{\Delta S}{\Delta f}.$$

Because we assume that the futures and spot prices will move in the same direction, ΔS and Δf have the same sign; thus, N_f is negative. This example therefore is a short hedge, but the concept is equally applicable to a long hedge, where $\Pi = -\Delta S + \Delta f N_f$ and N_f would be positive.

Now we need to know the ratio $\Delta S/\Delta f$. There are several approaches to estimating this value.

Minimum Variance Hedge Ratio

The objective of a hedge—indeed, of any investment decision—is to maximize the investor's expected utility. In this book, however, we do not develop the principles of expected utility maximization. Therefore, we shall take a much simpler approach to the hedging problem and focus on risk minimization. The model used here comes from the work of Johnson (1960) and Stein (1961).

The profit from the short hedge is[5]

$$\Pi = \Delta S + \Delta f N_f.$$

The variance of the profit is

$$\sigma_\Pi^2 = \sigma_{\Delta S}^2 + \sigma_{\Delta f}^2 N_f^2 + 2\text{Cov}_{\Delta S,\Delta f} N_f,$$

where

σ_Π^2 = variance of hedged profit

$\sigma_{\Delta S}^2$ = variance of change in the spot price

$\sigma_{\Delta f}^2$ = variance of change in the futures price

[4]Technically the hedge ratio is the dollar value of the futures position relative to the dollar value of the spot position. It is then used to determine the number of futures contracts necessary. In this book we shall let the hedge ratio refer to the number of futures contracts.

[5]The problem as formulated here is in terms of the profit. An alternative, and in some ways preferable, formulation is in terms of the rate of return on the hedger's wealth. We shall use the profit formulation here because it is more frequently seen in the literature.

$Cov_{\Delta S,\Delta f}$ = covariance of change in the spot price and change in the futures price

$\rho_{\Delta S,\Delta f}$ = correlation of the change in the spot price and change in the futures price.

Recall that $Cov_{\Delta S,\Delta f} = \sigma_{\Delta S,}\sigma_{\Delta f}\rho_{\Delta S,\Delta f}$; hence there is a direct relationship between covariance and correlation. The objective is to find the value of N_f that gives the minimum value of σ^2_{Π}.[6] The formula for the optimal number of futures contracts N_f^*, is

$$N_f^* = -\frac{Cov_{\Delta S,\Delta f}}{\sigma^2_{\Delta f}}$$

This formula gives the number of futures contracts that will produce the lowest possible variance. In nearly all cases, the covariance/correlation is positive so the negative sign means that the hedger should sell futures. If the problem were formulated as a long hedge, the sign would be positive.

You may recognize that the formula for N_f is very similar to that for β from a least squares regression. In fact, we can estimate N_f by running a regression with ΔS as the dependent variable and Δf as the independent variable. Of course, this can give us the correct value of N_f only for a historical set of data. We cannot know the actual value of N_f over an upcoming period. Extrapolating the future from the past is risky, but at least it is a starting point.

The effectiveness of the minimum variance hedge can be estimated by examining the percentage of the risk reduced. Suppose we define hedging effectiveness as

$$e^* = \frac{\text{Risk of unhedged position} - \text{Risk of hedged position}}{\text{Risk of unhedged position}}$$

This can be written as

$$e^* = \frac{\sigma^2_{\Delta S} - \sigma^2_{\Pi}}{\sigma^2_{\Delta S}}.$$

Thus, e* gives the percentage of the unhedged risk that the hedge eliminates. By substituting the formulas for σ^2_{Π} and N_f^* in the formula for e* and rearranging terms, we get

$$e^* = \frac{N_f^{*2}\sigma^2_{\Delta f}}{\sigma^2_{\Delta S}} = \rho^2_{\Delta S,\Delta f}.$$

This happens to be the formula for the coefficient of determination for the regression, which is the square of the correlation coefficient. It indicates the percentage of the variance in the dependent variable, ΔS, that is explained by the independent variable, Δf.

We shall see an example of the minimum variance hedge ratio in a hedging situation we examine later in this chapter.

[6]See Hedge Ratios and Futures Contracts Technical Note, in the section called "Derivation of Minimum Variance Hedge Ratio."

Price Sensitivity Hedge Ratio

The price sensitivity hedge ratio comes from the work of Kolb and Chiang (1981). The objective here is to determine the value of N_f^* that will result in minimum change in the bond portfolio's value for a small change in interest rates. Because the strategy is designed for interest rate futures, we will illustrate it with reference to a bond and a bond futures contract.

In order to understand the price sensitivity formula, we must first review the concept of a bond's duration. Duration has several specific definitions, but generally is used as a measure of price sensitivity. The bond price, B, is the sum of the present values of each of its cash payments—coupon interest and principal. These present values can be found by discounting each cash payment at a single interest rate, which is known as the yield or sometimes yield to maturity (y_B). Formally, we have

$$B = \sum_{t=1}^{T} \frac{CP_t}{(1 + y_B)^t},$$

where CP_t is the cash payment made at time t and will be either the coupon interest or principal. If the yield changes, we know that the price changes inversely. An approximation to the change in price as it relates to the change in yield is given by the formula,

$$\Delta B \approx -B \frac{DUR_B(\Delta y_B)}{1 + y_B},$$

where DUR_B represents the bond's duration and Δ represents the change in B or y_B. Formally, the duration is a weighted average of the time to each cash payment date and is specified in units of time. There are several versions of the concept of duration. This particular one, though often just called duration, is more precisely identified as Macaulay's duration, named after one of the first economists to derive it.

Macaulay's duration measures the timing and size of a bond's cash flows. Bonds with high coupons and short maturities have short durations, whereas bonds with low coupons and long maturity have long durations. A zero coupon bond has a duration equal to its maturity, whereas a 30-year bond might have a duration of 8 to 14 years. From the above equation, we see that duration is a measure of a bond's price sensitivity, with longer duration bonds being more sensitive. For comparative purposes, we can say that, for example, a 10-year zero coupon bond has the same price sensitivity as a 30-year bond with a duration of 10.

One important variation of Macaulay's duration is called modified duration, which measures the bond price change, adjusted for the level of yield. Specifically, modified duration is expressed as

$$MD_B = \frac{DUR_B}{1 + y_B} \approx -\frac{\Delta B / B}{\Delta y_B}.$$

In all material that follows, we shall use modified duration. Although modified duration has some limitations, it is a very useful concept in hedging. Knowing the modified duration of a bond, we can hedge the bond's price sensitivity by using knowledge of the modified duration of an appropriate futures contract and trading enough futures to offset the risk of the bond.[7] Now let us look at how a hedge can be designed.

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[7]Another potential duration measure is effective duration. The effective duration is a cash-flow-adjusted measure of risk based on estimates of the percentage change in bond price for a given change in yield, incorporating any cash flow effects of the given change in yield. For example, a callable bond may be called if interest rates fall, thus shortening the duration to zero when it is called.

Suppose changes in interest rates on all bonds are caused by a change in a single interest rate, r, which can be viewed as a default-free government bond rate. Thus, when r changes, all other bond yields change. Let B be the price of the bond held in the spot market and y_B be its yield. We will assume the bond futures contract is based on a single government bond. The futures contract has a price of f. Based upon that price, the remaining life of the deliverable bond at the expiration of the futures, and the coupon of the deliverable bond, we can infer that the deliverable bond at expiration would have a yield of y_f and a modified duration of MD_f. We shall refer to these as the implied yield and the implied modified duration of the futures.

The profit from a hedge is the change in value of the hedger's position as a result of a change in the rate, r. If we are hedged, this change in value is should be zero.[8] The formula for N_f^* is

$$N_f^* = -\left(\frac{\Delta B}{\Delta f}\right)\left(\frac{\Delta y_f}{\Delta y_B}\right).$$

This expression can be considerably simplified. Recall from the formula above that we can express modified duration as

$$MD_B \approx -\frac{\Delta B / B}{\Delta y_B}.$$

The implied modified duration of a futures contract can be expressed in a similar manner as

$$MD_f \approx -\frac{\Delta f / f}{\Delta y_f}.$$

Letting $\Delta y_f = \Delta y_B$ and substituting in the expression for N_f gives

$$N_f^* = -\left(\frac{MD_B}{MD_f}\right)\left(\frac{B}{f}\right).$$

Yet another version of the price sensitivity hedge ratio is often used in practice. It is given as

$$N_f^* = -(\text{Yield beta})\frac{PVBP_B}{PVBP_f},$$

where $PVBP_B$ is the present value of a basis point change for the bond and is specifically defined as $\Delta B/\Delta y_B$, which we know is $-MD_B B$. $PVBP_f$ is the present value of a basis point change for the futures and is defined as $\Delta f/\Delta y_f$, which is $-MD_f f$. These variables are, in effect, the change in the price of the bond or futures for a change in the yield of Δy_B or Δy_f.

The yield beta is the coefficient from a regression of the bond yield on the implied yield of the futures. In the price sensitivity formula, we assumed the bond yield changes one for one with the implied yield on the futures. This makes the yield beta 1. The yield beta, however, is not always equal to one and the preceding equation becomes

$$N_f^* = -\left(\frac{MD_B}{MD_f}\right)\left(\frac{B}{f}\right)\beta_y,$$

[8] See Hedge Ratios and Futures Contracts Technical Note, in the section called "Derivation of Price Sensitivity Hedge Ratio."

which is the yield-beta-adjusted price sensitivity formula. If the hedger does not believe the bond and futures prices will change in a one-to-one ratio, however, the yield beta actually should be estimated. In the examples used in this chapter, we shall assume the yield beta is 1.

The price sensitivity formula takes into account the volatility of the bond and futures prices. Thus, it incorporates information from current prices rather than regressing past bond prices on past futures prices. There are merits to both approaches, and in practice one approach sometimes may be more practical than the other. We shall illustrate both methods in some hedging examples later in the chapter.

There is another consideration in using the price sensitivity formula. Because it is derived from calculus, the formula hedges against only very small changes in interest rates. These changes must occur over a very short period of time. Once the interest rate changes and/or a period of time elapses, the modified durations of the bond and futures change and a new hedge ratio is required. This problem is much like the delta hedging we covered earlier in the Black-Scholes-Merton model. It is possible that many of our bond hedges, which are over periods of a few months at most, would not require major changes in the hedge ratio. Regardless, for our purposes here, we shall maintain the hedge ratio at the same value as obtained from the formula under the initial conditions, keeping in mind that the hedge will be less than perfect. If we desire a perfect hedge in practice, we would have to make adjustments.

In the example developed here, the instrument being hedged is an individual bond. In practice, it is often the case that the instrument being hedged will be a bond portfolio. Thus, the value of the underlying, B, will be the sum of the values of the component bonds. The modified duration, MD_B, will be the overall modified duration of the portfolio, which is a weighted average of the modified durations of the component bonds with each bond's weight given by its market value relative to the overall market value of the portfolio. The overall yield, y_B, is the overall discount rate for the portfolio that equates the present value of all of the cash flows for the portfolio to the market value of the portfolio. It is a complex weighted average of the yields of the component bonds. We shall not deal with how the overall yield is obtained. Any bond portfolio management software can easily obtain this number, so we shall assume that it is known. Though the price sensitivity formula will often be used at the portfolio level, the principles underlying it are exactly the same as we learned here for individual bonds, and its effectiveness will be just as great, if not greater.

Stock Index Futures Hedging

Because the price sensitivity hedge is not applicable to stock index futures, the minimum variance hedge usually is employed. Suppose we define ΔS as Sr_S, where r_S is the return on the stock, which is the percentage change in price. Then we define Δf as fr_f, where r_f is the percentage change in the price of the futures contract. Note that this is not the return on the futures contract. Because there is no initial outlay, there is no return on a futures contract. If we substitute Sr_S and fr_f for ΔS and Δf in the minimum variance formula for N_f^*, we get

$$N_f^* = -\left(\frac{S}{f}\right)\left(\frac{Cov_{r_S,r_f}}{\sigma^2_{r_f}}\right)$$

where Cov_{r_S,r_f} is the covariance between r_S and r_f, and σ^2_{rf} is the variance of r_f. If we run a regression of the percentage change in the spot price on the percentage change in the futures price, we obtain a regression coefficient we can call β_S. You may recognize this concept as similar to the beta from the Capital Asset Pricing Model. Is this the same beta? Not exactly, but since the futures contract is based on a market index, the beta should be somewhat close. In that case,

$$N_f = -\left(\frac{\beta_S}{\beta_f}\right)\left(\frac{S}{f}\right)$$

is the minimum variance hedge ratio for a stock index futures contract where β_s is the beta of the stock portfolio, and β_f is the beta of the futures contract.

The beta of the stock portfolio is always measured relative to some broad-based index. It is often assumed that if the index underlying the futures is a broad-based index, the futures beta in the above equation would be close to 1.0 and could be effectively ignored. In that case, the futures beta will indeed be close to one; it differs by only a small interest and dividend-related adjustment. But many portfolios are not based on broad-based market indices even though their betas are measured relative to broad-based indices. If the hedger used a futures contract that would be a good hedging instrument for this portfolio, he would need to know the beta of this futures contract relative to the broad-based index. Thus, for example, a portfolio might consist of slightly higher than average risk stocks and have a beta of 1.15. A futures contract on a portfolio of over-the-counter stocks might make a good hedge. Its beta might be 1.10. Thus, the ratio 1.15/1.10 = 1.045 would play a role in the calculation of the hedge ratio and would be quite different from the result that would be obtained by assuming a beta of 1.0 for the futures. Throughout this chapter we shall assume a futures beta of 1.0.

In addition, the hedging approach described here does not take into account dividends on the stocks in the portfolio. Dividends will affect the overall outcome of the transaction, but the uncertainty of dividends over most hedge horizons is fairly small and is not a risk most portfolio managers would worry about having to hedge.

We shall see some examples of stock index futures hedging later in the chapter.

HEDGING STRATEGIES

So far we have examined some basic principles underlying the practice of hedging. The next step is to illustrate how these hedges are executed. We shall look at some examples developed from a variety of economic and financial environments that illustrate several hedging principles.

The examples are divided into three groups: foreign currency hedging, intermediate- and long-term interest rate risk hedging, and stock hedging. One category is conspicuously absent from our list: hedging short-term interest rates. Indeed, the risk associated with short-term interest rates is one of the most visible risks in the financial markets. Short-term interest rate futures contracts do exist for the purpose of hedging this type of risk, which is largely faced by corporations in the course of borrowing and lending, and by banks as they borrow, lend, and engage in their dealership activities in over-the-counter derivatives. But, in reality, the actual use of short-term interest rate futures for hedging purposes is relatively minimal. In the United States, futures contracts on Federal funds and one-month LIBOR have relatively low trading volume.

Eurodollar futures are widely traded but rarely used by corporations for hedging interest rate risk. Corporations prefer the use of swaps and customized interest rate derivatives for this purpose. Banks are heavy users of Eurodollar futures, but they do so largely to hedge the swaps and other interest rate derivatives they use in their activities as dealers. We shall cover in Chapters 12 and 13 these instruments and their use by corporations in hedging, and by banks in managing, their over-the-counter positions.

Foreign Currency Hedges

Before getting into the details of foreign currency hedging, let us review a few concepts of foreign currency futures and forward contracts. Futures contracts are available in given sizes, indicated by the amount of the foreign currency. For example, in the United States, three of the most actively traded foreign currency futures contracts are the euro, available with a size of €125,000, the British pound, available in units of £62,500, and the Japanese yen, available in contracts covering ¥12,500,000. Of course, a hedger can transact in only a whole number of futures contracts; thus, some hedges will cover slightly more or slightly less than the size of the position being hedged.

Foreign currency futures prices are stated in the currency of the home country. Thus in the United States, the euro futures price might be quoted as \$1.05, and the British pound futures price might be \$1.58. For a single contract, this would mean that the aggregate price would be 125,000(\$1.05) = \$131,250 for the euro contract and 62,500(\$1.58) = \$98,750 for the pound contract. Because of the large number of Japanese yen in a dollar (typically over 100), the price is usually quoted without the two leading zeroes. Thus, a price of 0.8310 is really \$0.008310, which is equivalent to about 120.34 yen per dollar. The aggregate contract price would be 12,500,000 (\$0.008310) = \$103,875.

Long Hedge with Foreign Currency Futures Recall that a long hedge with futures involves the purchase of a futures contract. In the case of foreign currencies, a long hedger is concerned that the value of the foreign currency will rise. An example is presented in Table 11.5.

Here an American car dealer plans to buy 20 British sports cars. Each car costs 35,000 pounds, which will have to be paid in the British currency. Based on the current forward rate of the pound, the dealer's current cost is \$914,200. If, however, the pound increases in value, the cars will end up costing more. The dealer hedges by buying futures on the pound. As the table indicates, this was a good decision because the pound did appreciate; the cars ended up costing \$1,009,400, which is \$95,200 more, but the futures contracts generated a profit of \$109,656.25, which more than covered the increased cost of the cars.

Table 11.5 A Long Hedge with Foreign Currency Futures

Scenario: On July 1, an American auto dealer enters into a contract to purchase 20 British sports cars with payment to be made in British pounds on November 1. Each car will cost 35,000 pounds. The dealer is concerned that the pound will strengthen over the next few months, causing the cars to cost more in dollars.

Date	Spot Market	Futures Market
July 1	The current exchange rate is \$1.3190 per pound. The forward rate of the pound is \$1.3060. Forward cost of 20 cars: 20(35,000)(\$1.3060) = \$914,200.	December pound contract is at \$1.278. Price per contract: 62,500(\$1.278) = \$79,875. The appropriate number of contracts is $\frac{20(35,000)}{62,500} = 11.2.$ Buy 11 contracts
November 1	The spot rate is \$1.442. Buy the 700,000 pounds to purchase 20 cars. Cost in dollars: 700,000(\$1.442) = \$1,009,400.	December pound contract is at \$1.4375. Price per contract: 62,500(\$1.4375) = 89,843.75. Sell 11 contracts

Analysis: The cars ended up costing \$1,009,400 − \$914,200 = \$95,200 more.

The profit on the futures transaction is

11(\$89,843.75)	(sale price of futures)
−11(\$79,875)	(purchase price of futures)
\$109,656.25	(profit on futures).

The profit on the futures more than offsets the higher cost of the cars, leaving a net gain of \$109,656.25 − \$95,200 = 4,456.25. The dealer effectively paid \$1,009,400 − \$109,656.25 = \$899,743.75 for the 20 cars.

As long as the pound spot and futures rates move in the same direction, the hedge will be successful in reducing some of the loss in the spot market. Had the pound weakened, there would have been a loss in the futures market that would have offset some or all of the gain in the spot market.

This transaction was executed in the futures market. Remember that futures contracts have standardized terms and provide a guarantee against default on the part of the other party. Forward contracts permit the parties to customize the terms of the contract. This transaction, however, would not be large enough to justify a forward contract. In the next example we shall execute the hedge in the forward market.

Short Hedge with Foreign Currency Forwards A short hedge is a commitment to sell a currency using futures or forwards and is designed to protect against a decrease in the foreign currency's value. In Table 11.6 we have a situation in which a multinational firm has 10 million pounds that it will convert at a later date. A transaction of this size can often be better executed with a customized forward contract. In addition, when the amount of exposure is large, the basis risk of a futures contract may be a risk not worth bearing. In this example the customer is long pounds and is exposed to the risk of the pound weakening. To protect against this risk, it sells a forward contract on the exact number of pounds it will convert, with the contract expiring on the day of conversion. This allows the firm to convert the pounds to dollars by simply delivering them to the dealer on the opposite side of the forward contract. As the table indicates, the pound did depreciate, which would have caused a loss of $1,195,000. Instead the firm locked in the rate on the day it entered into the forward contract.

In Chapter 12, we shall return to the subject of foreign currency hedging, when we learn about currency swaps. For now, let us turn to the hedging of intermediate- and long-term interest rate risk.

Table 11.6 A Short Hedge with Foreign Currency Forwards

Scenario: On June 29, a multinational firm with a British subsidiary decides it will need to transfer 10 million pounds from an account in London to an account with a New York bank. Transfer will be made on September 28. The firm is concerned that over the next two months the pound will weaken.

Date	Spot Market	Forward Market
June 29	The current exchange rate is $1.362 per pound. The forward rate of the pound is $1.357. Forward value of funds: 10,000,000($1.357) = $13,570,000.	Sell pounds forward for delivery on Septembe 28 at $1.357.
September 28	The spot rate is $1.2375.	Deliver pounds and receive 10,000,000($1.357) = $13,570,000.

Analysis: The pounds end up worth $13,570,000 − $12,375,000 = $1,195,000 less but are delivered on the forward contract for $13,570,000, thus completely eliminating the risk. Had the transaction not been done, the firm would have converted the pounds at the spot rate of $1.2375.

Intermediate- and Long-Term Interest Rate Hedges

The risk associated with intermediate- and long-term interest rates is typically faced by bond portfolio managers, who are responsible for current and anticipated positions in bonds. These bonds can be those issued by governments or corporations. The interest rate futures markets in most countries are extremely active and are widely used by bond portfolio managers to hedge the risk associated with interest rate changes. Most of these futures contracts are based on federal government bonds, which are usually more actively traded than corporate or non-federal government bonds. We should note that many risks related to corporate, state, and non-federal government bonds are a function of changes in perceptions of credit quality. The hedging of credit risk is addressed in Chapter 15.

We now turn to a look at some examples of bond portfolio hedging, employing the U.S. Treasury note and bond contracts of the Chicago Board of Trade. Treasury note and bond contracts on the Chicago Board of Trade are virtually identical except that there are three T-note contracts that are based on 2-year, 5-year, and 10-year maturities whereas the T-bond contract is based on Treasury bonds with maturities of at least 15 years that are not callable for at least 15 years. Thus, the T-note contracts are intermediate-term interest rate futures contracts and the T-bond contract is a long-term interest rate futures contract. Other than the difference in maturity of the underlying instruments and the margin requirements, the contract terms are essentially identical.

As discussed in Chapter 10, the T-bond contract is based on the assumption that the underlying bond has a coupon rate of 6 percent and, as mentioned, a maturity or call date of not less than 15 years. The specific coupon requirement is not restrictive, however; the CBOT permits delivery of bonds with other coupon rates, with an appropriate adjustment (recall the conversion factor) made to the price received for the bonds. There are many different bond issues eligible for delivery on a given contract.

Recall that T-bond futures prices are quoted in dollars and thirty-seconds of par value of $100. For example, a futures price of 93-14 is 93 14/32, or 93.4375. The face value of T-bonds underlying the contract is $100,000; therefore, a price of 93.4375 is actually $93,437.50. Expiration months are March, June, September, and December, extending out about two years. The last trading day is the business day prior to the last seven days of the expiration month. The first delivery day is the first business day of the month.

DERIVATIVES TOOLS

Concepts, Applications, and Extensions
Hedging Contingent Foreign Currency Risk

In this chapter we are learning how to use forward and futures contracts to hedge. If the hedger wants to receive the benefits of favorable movements in the underlying, options can be used. We saw an example of this type of strategy in Chapter 6 with the use of covered calls and protective puts.

There is one particular situation in which currency options can be particularly valuable: the hedging of contingent foreign currency risk. This scenario occurs when a party anticipates the possibility—but is not certain—of a future position in a foreign currency.

For example, suppose an American firm is bidding for a contract to construct a sports complex in London. The bid must be submitted in British pounds. The firm plans to make a bid of £25 million. At the forward exchange rate of $1.437, the bid in dollars is equivalent to £25,000,000 ($1.437) = $35,925,000. Once the bid is submitted, the firm must be prepared to accept £25 million if the bid is successful. Because it is an American firm, it will convert the pounds into dollars at whatever rate prevails on the date payment is made. If the pound weakens, the firm will effectively receive fewer dollars. To simplify the example somewhat, we shall assume the payment will be made as soon as the decision is made as to which firm is awarded the construction contract.

Consider the possibility of a forward or futures hedge in comparison to the purchase of a put on the pound. For discussion purposes, we shall refer only to the forward and put hedges.

If the bid is successful and the pound increases, the firm will receive the pounds, which now are valued at more dollars per pound. The forward hedge will, however, reduce this gain, because the hedge will be a short position. If the option is used, the put will expire worthless.

If the bid is successful and the pound decreases, the forward hedge will reduce the loss caused by the decline in the pound's value. The option will, however, also reduce the loss on the pound.

If the bid is unsuccessful and the pound increases, the forward hedge will result in a potentially large speculative loss. This is because the firm will not receive the pounds if the bid fails but will have a short position in a forward contract. If the option hedge is used, the put will expire worthless. The firm will have lost money— the premium on the put—but the amount lost is likely to be less than it would have been with the forward hedge.

If the bid is unsuccessful and the pound decreases, the forward hedge will result in a potentially large speculative profit, because the firm will be short forward and will not receive the pounds as a result of the failure to win the bid. If the option hedge is used, the put's exercise will also result in a potentially large profit on the put.

The option hedge is most beneficial when the bid is unsuccessful. Because the firm does not receive the pounds, the forward position generates a potentially large profit or loss. The option, however, can generate a large profit if the pound declines; if the pound rises, the loss will be limited to the premium. Of course, the option hedge requires payment of the option premium, while the forward hedge might require collateral. Neither type of hedge dominates the other, but each has its merits.

Let us now look at this specific hedge. The firm needs to hedge the anticipated receipt of £25 million. We shall assume it uses either forwards or over-the-counter put options. The forward price is $1.424 and the option premium is $0.025. Thus, the option will cost £25,000,000 ($0.025) = $625,000. The forward contract locks in £25,000,000($1.424) = $35,600,000. We can assume that the option or forward expires on the date on which the outcome of the bid is determined and that on that date the firm either receives the cash payment of £25 million or not.

For the case in which the bid is successful, either hedge works fairly well. The option hedge is like a protective put, which benefits from a strong pound and which has limited losses from a weak pound. The forward hedge locks in $35,600,000 regardless of the value of the pound. If the firm does not hedge, note that the overall value moves one-for-one with the value of the pound.

For the case in which the bid is unsuccessful, the forward hedge leaves the firm highly exposed, with the potential for a rather substantial loss. In the option hedge, the most the firm can lose is the option premium of $625,000, but it can gain if the pound decreases, even though it did not win the bid.

Of course, the firm could choose not to bid, but this is unlikely because bidding on contracts is the nature of the construction business. The firm could choose not to hedge, but it could win the bid and earn a much smaller profit or even a loss if the pound falls significantly. The option hedge provides an alternative that will be attractive to some firms, while the forward hedge will be better for others. The differences in their expectations and willingness to take exchange rate risk will determine whether they use options or forwards.

Hedging a Long Position in a Government Bond Portfolio managers constantly face decisions about when to buy and sell securities. In some cases, such decisions are automatic. Securities are sold at certain times to generate cash for meeting obligations, such as pension payments. Consider the following example.

On February 25, a portfolio manager holds $1 million face value of government bonds with a coupon of 11 7/8 percent and maturing in about 25 years. The bond currently is priced at 101 per $100 par value, and the yield is 11.74 percent. The modified duration is 7.83 years. The bond will be sold on March 28 to generate cash to meet an obligation.[9]

The portfolio manager is concerned that interest rates will increase, resulting in a lower bond price and the possibility that the proceeds from the bond's sale will be inadequate for meeting the obligation. The manager knows that if interest rates increase, a short futures position will yield a profit that can offset at least part of any decrease in the bond's value. Since this is a government bond, the Treasury bond futures contract should be used.[10]

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[9]In actual situations a portfolio manager would hold a diversified portfolio of bonds. If the intention were to hedge the entire portfolio, the hedge ratio would be based on the overall portfolio value, yield, and duration. We assume the manager needs approximately $1 million of cash on March 28 and will sell only this bond to generate the cash.

[10]Technically, the accrued interest would be a component of the outcome of this strategy, but it is not subject to any uncertainty so we leave it out of the hedging examples.

Table 11.7 presents the results of the hedge. The manager will use the June T-bond futures contract. Using the price sensitivity hedge ratio, the manager determines that he should sell 16 contracts. When the bonds were sold on March 28 they generated a loss of over $53,000. The futures transaction produced a profit of over $60,000. Thus, the hedge eliminated all the loss and even produced a gain. Had bond prices moved up, the futures price would have increased and the futures transaction would have generated a loss that would have reduced or perhaps eliminated all of the increase in the value of the bonds.

This short hedge represents one of the most common hedging applications, and we shall see a slight variation of it later when we examine stock index futures hedging. This hedge is applicable to many firms and institutions, such as banks, insurance companies, pension funds, and mutual funds.

Table 11.7 Hedging a Long Position in a Government Bond

Scenario: On February 25, a portfolio manager holds $1 million face value of a government bond, the 11 7/8s, which mature in about 25 years. The bond is currently priced at 101 and has a modified duration of 7.83. The manager will sell the bond on March 28 to generate cash to meet an obligation.

Date	Spot Market	Futures Market
February 25	The current price of the bonds is 101. Value of position: $1,010,000. The short end of the term structure is flat, so this is the forward price of the bonds in March.	June T-bond futures is at 70 16/32. Price per contract: $70,500. The futures price and the characteristics of the deliverable bond imply a modified duration of 7.20. Appropriate number of contracts: $N_f = -\left(\frac{7.83}{7.20}\right)\left(\frac{1{,}010{,}000}{70{,}500}\right)$ $= -15.6.$ **Sell 16 contracts**
March 28	The bonds are sold at the current price of 95 22/32. This is a price of $956.875 per bond. Value of position: $956,875.	June T-bond futures is at 66 22/32. Price per contract: $66,718.75. **Buy 16 contracts**

Analysis: When the $1 million face value bonds are sold on March 28, they are worth $956,875, a loss in value of $1,010,000 – $956,875 = $53,125.

The profit on the futures transaction is

$$\begin{array}{rl} 16(\$70{,}500) & \text{(sale price of futures)} \\ -\ 16(\$66{,}718.75) & \text{(purchase price of futures)} \\ \hline \$60{,}500 & \text{(profit on futures).} \end{array}$$

Thus, the hedge eliminated the entire loss in value and resulted in an overall gain in value of $60,500 – $53,125 = $7,375.

Anticipatory Hedge of a Future Purchase of a Treasury Note Previously we saw how one could hedge the future purchase of a Treasury bond. In this example, we do the same with a Treasury note.

Suppose that on March 29, a portfolio manager determines that approximately $1 million will be available on July 15. The manager decides to purchase the 11 5/8 Treasury notes maturing in about nine years. The forward price of the notes is 97 28/32, or $978,750, for $1 million face value. This price implies a forward yield of 12.02 percent. If yields decline, the notes' price will increase and the manager may be unable to make the purchase. If this happens, a profit could have been made by purchasing futures contracts. Because the Treasury note futures contract is quite liquid, the manager decides to buy T-note futures. Because the hedge is to be terminated on July 15, the September contract is appropriate. The results are presented in Table 11.8.

When the bonds are ultimately purchased, they end up costing over $97,000 more, but the futures transaction generated a profit of over $82,000. Thus, the effective purchase price is actually about $15,000 higher and gives an effective yield of 11.75 percent, which is reasonably close to the target forward yield of 12.02 percent.

Had bond prices moved down, the hedger would have regretted doing the hedge. The notes would have cost less, but this would have been offset by a loss on the futures contract. Once again, this is the price of hedging—forgoing gains to limit losses.

Hedging a Corporate Bond Issue One interesting application of an interest rate futures hedge occurs when a firm decides to issue bonds at a future date. There is an interim period during which the firm prepares the necessary paperwork and works out an underwriting arrangement for distributing the bonds. During that period, interest rates could increase so that when the bonds ultimately are issued, they will command a higher yield. This will be more costly to the issuer.

Table 11.8 Anticipatory Hedge of a Future Purchase of a Treasury Note

Scenario: On March 29, a portfolio manager determines that approximately $1 million will be available for investment on July 15. The manager plans to purchase the 11 5/8s Treasury notes maturing in about nine years, which have a modified duration of 5.6.

Date	Spot Market	Futures Market
March 29	The forward price of notes is 97 28/32. Current forward value of notes: $978,750. This implies a yield of 12.02 percent.	September T-note futures are at 78 21/32. Price per contract: $78,656.25. The futures price and the characteristics of the deliverable bond imply a modified duration of 6.2. Appropriate number of contracts: $N_f = -\left(\frac{\$978,750}{\$78,656.25}\right)\left(\frac{5.6}{6.2}\right) = 11.24.$ **Buy 11 contracts**
July 15	The notes are purchased at their current price of 107 19/32. This is a price of $1,075.9375 per note. Value of position: $1,075,937.50.	September T-note futures is at 86 6/32. Price per contract: $86,187.50. **Sell 11 contracts**

Analysis: When the $1 million face value notes are purchased on July 15, they cost $1,075,937.50, an increased cost of $1,075,937.50 – $978,750 = $97,187.50. The yield at this price is 10.31 percent.

The profit on the futures transaction is

11($86,187.50) (sale price of futures)
– 11($78,656.25) (purchase price of futures)
$82,843.75 (profit on futures).

Thus, the hedge offset about 85 percent of the increased cost. The effective purchase price of the notes is $1,075,937.50 – $82,843.75 = $993,093.75, which is an effective yield of 11.75 percent.

Consider the following example. On February 24, a corporation decides to issue $5 million face value of bonds on May 24. As a standard of comparison, the firm currently has a bond issue outstanding with a coupon of 9 3/8 percent, a yield of 13.76 percent, and a maturity of about 21 years. Any new bonds issued will require a similar yield. Thus, the firm expects that when the bonds are issued in May, the coupon would be set at 13.76 percent, so the bonds would go out at par.

If rates increase, the firm will have to discount the bonds or adjust the coupon upward to the new market yield. We shall assume that the coupon is fixed so that the price will decrease. In either case, the firm will incur a loss. The firm realizes that if rates increase, it can make a profit from a short transaction in futures. Thus, it decides to hedge the issue by selling futures contracts.

There is no corporate bond futures contract, so the hedger chooses the Treasury bond futures contract. Because the hedge will be closed on May 24, the June contract is chosen.

The hedge is illustrated in Table 11.9. Using the price sensitivity hedge ratio, the firm sells 67 futures contracts. When the bonds are ultimately issued, the yield is 15.25 percent. This results in a loss of over \$460,000 on the bonds, but the futures transaction made over \$500,000. The firm made a net gain of about \$44,000. The futures profit can be added to the proceeds from the bond of about \$4.5 million so as to infer an effective issue price of slightly more than \$5 million. This sets the effective yield on the bonds at 13.63 percent, which is quite close to the target of 13.76 percent.

Table 11.9 Hedging a Corporate Bond Issue

Scenario: On February 24, a corporation decides to issue \$5 million of bonds on May 24. The firm currently has outstanding comparable bonds with a coupon of 9 3/8, a yield of 13.76 percent, and a maturity of about 21 years. The firm anticipates that if conditions do not change, the bonds when issued in May will be issued with a 13.76 percent coupon and be priced at par with a 20-year maturity and a modified duration of 7.22.

Date	Spot Market	Futures Market
February 24	If issued in May, it is expected that the bonds would offer a coupon of 13.76 percent and be priced at par with a modified duration of 7.22. Value of position: \$5,000,000.	June T-bond futures are at 68 11/32. Price per contract: \$68,343.75. The futures price and the characteristics of the deliverable bond imply a modified duration of 7.88 and a yield of 13.60 percent. Appropriate number of contracts: $N_f = -\left(\frac{7.22}{7.88}\right)\left(\frac{5,000,000}{68,343.75}\right)$ $= -67.0.$ **Sell 67 contracts**
May 24	The yield on comparable bonds is 15.25 percent. The bonds are issued with a 13.76 percent coupon at a price of 90.74638. Price per bond: \$907.46. Value of bonds: \$4,537,319.	June T-bond futures is at 60 25/32. Price per contract: \$60,781.25. **Buy 67 contracts**

Analysis: When the \$5 million face value bonds are issued on May 24, they are worth \$4,537,319, a loss in value of \$5,000,000 − \$4,537,319 = \$462,681. The yield at this price is 15.25 percent. (Note: Alternatively, if the bonds actually were issued at par with the coupon set at the yield on May 24 of 15.25 percent, the firm would receive the full \$5,000,000 but the present value of its increased interest cost would be \$462,681.)

The profit on the futures transaction is

67(\$68,343.75) (sale price of futures)
− 67(\$60,781.25) (purchase price of futures)
\$506,687.50 (profit on futures).

Thus, the hedge offset more than all of the increased cost and left a net gain of \$506,687.50 − \$462,681 = \$44,006.50. The effective issue price of the bonds is \$4,537,319 + \$506,687.50 = \$5,044,006.50, which is an effective yield of 13.63 percent.

Had interest rates declined, the firm would have obtained a higher price for the bonds; however, this would have been at least partially offset by a loss on the futures transaction. By executing the hedge, the firm was able to protect itself against an interest rate change while preparing the issue. In a similar vein, investment bankers might do this type of hedge. An investment banker purchases the bonds from the firm and then resells them to investors. Between the time the bonds are purchased and resold, the investment banker is exposed to the risk that bond yields will increase.[11] Therefore, a short hedge such as this would be appropriate. Many investment banking firms have been able to protect themselves against large losses by hedging with interest rate futures.

Forward contracts are less commonly used than futures to hedge the risk associated with intermediate- and long-term securities. The futures markets have developed a niche in meeting the hedging needs of parties bearing this kind of risk.

Stock Market Hedges Stock index futures have been one of the spectacular success stories of the financial markets. These cash-settled contracts are based on stocks. Investors use them to hedge positions in stock, speculate on the direction of the stock market in general, and arbitrage the contracts against comparable combinations of stocks.

Stock index futures contracts are based on indices of common stocks. The most widely traded contract in the United States is the S&P 500 futures at the Chicago Mercantile Exchange. The futures price is quoted in the same manner as the index. The futures contract, however, has an implicit multiplier of $250. Thus, if the futures price is 1,300, the actual price is 1,300($250) = $325,000. At expiration, the settlement price is set at the price of the S&P 500 index and the contract is settled in cash. The expirations are March, June, September, and December. The last trading day is the Thursday before the third Friday of the expiration month.

Several of the hedging examples illustrated with T-note and T-bond futures are similar to stock index futures hedges where a firm attempts to hedge a long position in a security or portfolio. The first example we shall consider is the hedge of a stock portfolio.

Stock Portfolio Hedge A central tenet of modern investment theory is that diversification eliminates unsystematic risk, leaving only systematic risk. Until the creation of stock index futures, investors had to accept the fact that systematic risk could not be eliminated. Now investors can use stock index futures to hedge the systematic risk. But should they do that? If all systematic and unsystematic risk is eliminated, the portfolio can expect to earn only the risk-free return. Why not just buy risk-free bonds? The answer is that investors occasionally wish to change or eliminate systematic risk for brief periods. During times of unusual volatility in the market, they can use stock index futures to adjust or eliminate the systematic risk. This is much easier and less costly than adjusting the relative proportions invested in each stock. Later the portfolio manager can close out the futures position, and the portfolio systematic risk will be back at its original level.

Consider a portfolio manager who on March 31 is concerned about the market over the period ending July 27. The portfolio has accumulated an impressive profit, and the manager would like to protect the portfolio value over this time period. The manager knows that the portfolio is exposed to a loss in value resulting from a decline in the market as a whole, the systematic risk effect, as well as losses resulting from the unsystematic risk of the individual stocks. Although the portfolio contains only eight stocks, the manager is not particularly worried about the unsystematic risk. The manager knows that the systematic risk can be hedged by using S&P 500 stock index futures, specifically the September contract, which we shall assume has a beta of 1.0.

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[11]Investment bankers do employ other means of minimizing their risk exposure. The use of a syndicate, in which a large number of investment bankers individually take a small portion of the issue, spreads out the risk. Many issues are taken on a "best efforts" basis. This allows the investment banker to return the securities to the issuing firm if market conditions make the sale of the securities impossible without substantial price concessions.

The results are shown in Table 11.10. The portfolio beta, which reflects the influence of the market as a whole on a stock or portfolio, is a weighted average of the betas of the component stocks, where each weight of a given stock is the market value of that stock divided by the total market value of the portfolio.

Table 11.10 Stock Portfolio Hedge

Scenario: On March 31, a portfolio manager is concerned about the market over the next four months. The portfolio has accumulated an impressive profit, which the manager wishes to protect over the period ending July 27. The prices, number of shares, and betas are given below:

Stock	Price (3/31)	Number of Shares	Market Value	Weight	Beta
Federal Mogul	18.875	18,000	$339,750	0.044	1.00
Lockheed Martin	73.500	16,000	1,176,000	0.152	0.80
IBM	50.875	7,000	356,125	0.046	0.50
US West	43.625	10,800	471,150	0.061	0.70
Bausch & Lomb	54.250	21,000	1,139,250	0.147	1.10
First Union	47.750	28,800	1,375,200	0.178	1.10
Walt Disney	44.500	25,000	1,112,500	0.144	1.40
Tesoro	52.875	33,200	1,755,450	0.227	1.20
			$7,725,425	1.000	

Portfolio beta:

$0.044(1.00) + 0.152(0.80) + 0.046(0.50) + 0.061(0.70) + 0.147(1.10) + 0.178(1.10) + 0.144(1.40) + 0.227(1.20) = 1.06.$

S&P 500 September futures contract (assumed to have a beta of 1.0):

Price on March 31: 1305

Multiplier: $250

Price of one contract: $250(1305) = $326,250.

Optimal number of futures contracts:

$$N_f = -1.06\left(\frac{7,725,425}{326,250}\right) = -25.10.$$

Sell 25 contracts

Stock	Price (7/27)	Market Value
Federal Mogul	21.625	$389,250
Lockheed Martin	81.500	1,304,000
IBM	43.875	307,125
US West	47.125	508,950
Bausch & Lomb	45.875	963,375
First Union	48.125	1,386,000
Walt Disney	40.000	1,000,000
Tesoro	50.000	1,660,000
		$7,518,700

Results: The values of the stocks on July 27 are shown below:

S&P 500 September futures contract:

Price on July 27: 1,295.40

Multiplier: $250

Price of one contract: $250(1,295.40) = $323,850.

Buy 25 contracts

Analysis: The market value of the stocks declined by $7,725,425 – $7,518,700 = $206,725, a loss of 2.68 percent.

The futures profit was

25($326,250)	(sale price of futures)
–25($323,850)	(purchase price of futures)
$60,000	(profit on futures).

Thus, the overall loss on the stocks was effectively reduced to $206,725 – $60,000 = $146,725, a loss of 1.90 percent.

This gives a portfolio beta of 1.06. Using the minimum variance hedge ratio results in the sale of 25 contracts. On July 27, the portfolio has declined in value by over $200,000, a loss of 2.68 percent. The futures transaction, however, generated a profit of $60,000, which reduced the effective loss to only 1.90 percent.

The objective of the hedge was to eliminate systematic risk. Clearly systematic risk was reduced but not eliminated. The stock portfolio value declined about 2.68 percent while the futures price decreased a little over 1 percent. The hedge certainly helped but was far from perfect. There are several possible explanations for this result. One is that the betas are an estimate taken from a popular investment advisory service. Beta estimates over the recent past have not necessarily been stable. It is also possible that the portfolio was not sufficiently diversified and some unsystematic risk contributed to the loss. Some of the stocks may have paid dividends during the hedge period. We did not account for these dividends in illustrating the hedge results. Dividends would have reduced the loss on the portfolio and made the hedge more effective.

Had the market moved up, the portfolio would have shown a profit, but this would have been at least partially offset by a loss on the futures transaction. In either outcome, however, the portfolio manager would have been reasonably successful in capturing at least some of the accumulated profit on the portfolio.

Anticipatory Hedge of a Takeover or Acquisition The exciting world of takeovers and acquisitions offers an excellent opportunity to apply hedging concepts. The acquiring firm identifies a target firm and intends to make a bid for the latter's stock. Typically the acquiring firm plans to purchase enough stock to obtain control. Because of the large amount of stock usually involved and the speed with which takeover rumors travel, the acquiring firm frequently makes a series of smaller purchases until it has accumulated sufficient shares to obtain control. During the period in which the acquiring firm is slowly and quietly buying the stock, it is exposed to the risk that stock prices in general will increase. This means that either the shares will cost more or fewer shares can be purchased.

Consider the following situation. On July 15, a firm has identified Helix Technology Corporation as a potential acquisition.[12] Helix stock currently is selling for $26.50 and has a beta of 1.80. The acquiring firm plans to buy 100,000 shares, which will cost $2.65 million. The purchase will be made on August 15. This could be viewed as one purchase in a series of purchases designed to ultimately acquire controlling interest in the target firm. The acquiring firm realizes that if stock prices as a whole increase, the shares will be more expensive. If the firm purchases stock index futures, however, any general increase in stock prices will lead to a profit in the futures market.

Because the hedge will be terminated on August 15, the acquiring firm chooses to buy September S&P 500 futures. Table 11.11 shows the results of the hedge. On August 15, the Helix stock price is $28.75. The shares thus cost an additional $225,000.[13] The profit on the futures transaction, however, was $250,125. Thus, the effective cost of the shares is $26.25.

The hedge was successful in reducing some of the additional cost of the shares; however, the unsystematic risk cannot be hedged. In takeover situations, the unsystematic risk is likely to be very high. For example, if word leaks out that someone is buying up the stock, the price will tend to rise substantially. This can occur even if the market as a whole is going down. Also, federal regulations require that certain takeover attempts be announced beforehand. If there were options or futures on the target firm's stock, however, the acquiring firm could use these to hedge the unsystematic risk.

The takeover game is intense and exciting, with high risk and the potential for large profits. Stock index futures can play an important role, but the extent to which futures are used to hedge this kind of risk is not known, because much of this kind of activity is done with a minimum of publicity.

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[12]The choice of Helix is merely for illustrative purposes. Helix is not known to be a takeover target as of this writing.

[13]It is unlikely that all of the 100,000 shares could have been purchased at the same price. Therefore, we should treat $28.75 as the average price at which the shares were acquired.

Table 11.11 Anticipatory Hedge of a Takeover

Scenario: On July 15, a firm has decided to begin buying up shares of Helix Technology Corporation with the ultimate objective of obtaining controlling interest. The acquisition will be made by purchasing lots of about 100,000 shares until sufficient control is obtained. The first purchase of 100,000 shares will take place on August 15. The stock is currently worth \$26.50 and has a beta of 1.80.

Date	Spot Market	Futures Market
July 15	Current price of the stock is 26.50. Current cost of shares: 100,000(\$26.50) = \$2,650,000. The beta is 1.80.	September S&P 500 futures is at 1,260.50. Price per contract: \$315,125. Approximate number of contracts: $N_f = -1.8\left(\frac{2,650,000}{315,125}\right) = -15.14.$ Since it should buy futures, ignore the negative sign. Buy 15 contracts
August 15	The stock is purchased at its current price of 28.75. Cost of shares: 100,000(\$28.75) = \$2,875,000.	September S&P 500 futures is at 1,327.20. Price per contract: \$331,800. Sell 15 contracts

Analysis: When the 100,000 shares are purchased on August 15, they cost \$2,875,000, an additional \$225,000.

The profit on the futures transaction is

15(\$331,800) (sale price of futures)
−15(\$315,125) (purchase price of futures
\$250,125 (profit on futures).

Thus, the hedge eliminated all of the additional cost and left a small gain. The shares end up effectively costing (\$2,875,000 − \$250,125)/100,000 = 26.25.

The takeover example is but one type of situation wherein a firm can use a long hedge with stock index futures. Any time someone is considering buying a stock, there is the risk that the stock price will increase before the purchase is made. Stock index futures cannot hedge the risk that factors specific to the company will drive up the stock price, but they can be used to protect against increases in the market as a whole.

Forward markets for stocks and stock indices are not widely used. As we shall see in Chapter 12, however, swaps, which are closely related to forward contracts, are popular tools for controlling stock market risk. We turn now to explore various spread strategies. Hedging strategies are focused on risk reduction, whereas spread strategies are typically focused on revenue enhancement with some incremental increase in risk.

SPREAD STRATEGIES

We turn now to selected spread strategies. Spread strategies are similar to hedge strategies in that there are two positions with one intended to offset or partially offset the risk of the other. In the case of a hedge, one position is in the underlying and one is in futures. In the case of a spread, both positions are in the futures. Two forms of spread strategies are reviewed, intramarket spreads and intermarket spreads. Intramarket spreads involve going long and short futures contracts on the same underlying instruments, whereas intermarket spreads involve going long and short futures contracts on different underlying instruments.

Spread strategies are very important to the liquidity of various futures contracts. Hedgers often have specific contracts and specific maturities required to achieve their hedging objectives. When arbitrage is feasible, arbitrage traders can provide the necessary liquidity to support hedging demand. Recall that arbitrage trading requires that the risk acquired is offset in a related instrument, often the underlying. Arbitrageurs may not be able to execute transactions in the underlying with sufficient volume. Arbitrages may be able to achieve their desired result with other futures contracts, resulting in spread strategies. Speculative traders and other market participants often use spread strategies to achieve their desired objectives. This additional trading activity often provides liquidity to support hedging demand. Speculators are very careful in their assessment of relative value, and for a sufficient risk premium, will be induced to take positions. We explore, first, selected intramarket spreads, and then turn our attention to selected intermarket spreads.

Intramarket Spreads

Intramarket spread trading activities add liquidity to the market, thereby making hedging activities less costly. We focus first on generic futures spreads using the carry arbitrage model, and then illustrate intramarket spreads using Treasury bond futures contracts and stock index futures contracts.

Generic Intramarket Futures Spreads We explore first the behavior of generic futures contracts. Recall from Chapter 9 that the futures price was established as the spot price plus the cost of carry. Now consider two futures contracts with different expirations, times T_1 and T_2. The longer-term contract has a futures price of $f_0(2)$, equal to $S_0 + \theta_{0,2}$, and the shorter-term contract has a futures price of $f_0(1)$, equal to $S_0 + \theta_{0,1}$. Subtracting the two, we obtain

$$f_0(2) - f_0(1) = \theta_{0,2} - \theta_{0,1}.$$

This equation defines the spread between futures prices. The spread between the nearby and deferred contracts is the difference in their respective costs of carry. The term $\theta_{0,2} - \theta_{0,1}$ is the cost of the carry for the time interval between T_1 and T_2, observed at time 0, based on the carry arbitrage relationship, $f_0(T_i) = S_0 + \theta_{0,i}, i = 1,2$. Therefore, the profit from a spread trade can be expressed as the profit on the two positions. Assuming we are long the T_2 contract and short the T_1 contract, the profit is

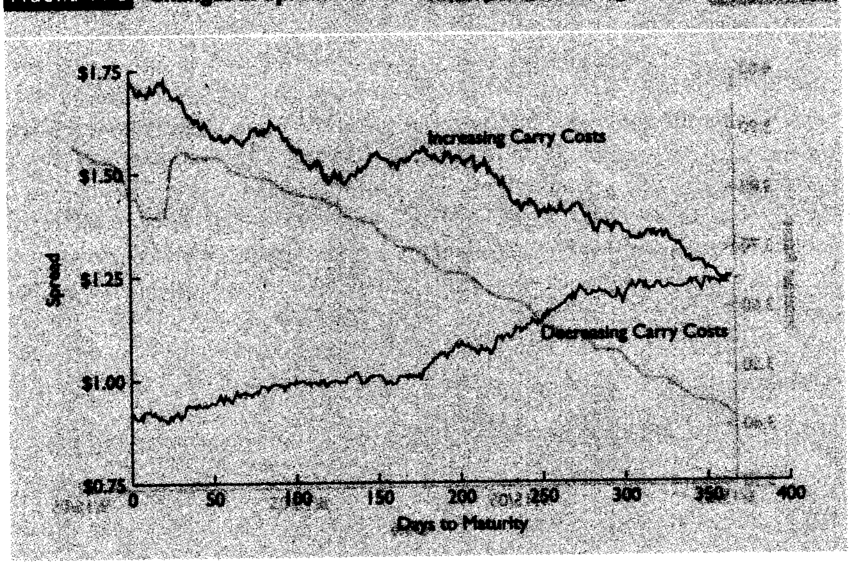
$$\Pi = [f_t(2) - f_0(2)] - [f_t(1) - f_0(1)]$$
$$= [\theta_{t,2} - \theta_{0,2}] - [\theta_{t,1} - \theta_{0,1}] = [\theta_{t,2} - \theta_{t,1}] - [\theta_{0,2} - \theta_{0,1}],$$

which is the difference between the change in carry costs of contracts 2 and 1. Based on this result, the risk and opportunities related to spread trading depend solely on changes in carry costs over time.

Successful spread trading will depend on successfully predicting changes in carry costs. Figure 11.2 illustrates the change in the spread based on a simulation of changes in carry costs.[14] Two cases are presented, one where carry costs are simulated to increase and the other where carry costs are simulated to decrease. We see that profitable spread trading strategies exist if one can reasonably predict changes in carry costs. If carry costs are expected to increase, then the trader should go long the more distant futures contract, T_2, and go short the nearby futures contract T_1. If carry costs are expected to decrease, then the trader should go short the more distant futures contract, T_2, and go long the nearby futures contract T_1.

[14]A generic carry arbitrage model is assumed with carry costs following geometric Brownian motion with mean of plus or minus 25 percent and a standard deviation of 10 percent.

FIGURE 11.2 Changes in Spread Based on Carry Costs Changes



Treasury Bond Futures Spreads Recall that Treasury bond futures contracts had several complicating issues, including the application of conversion factors and the question of which bond is the cheapest-to-deliver. Therefore, Treasury bond spread trading has some additional risks.

Treasury bond futures contracts are quoted in 32nds, hence the prices will exhibit jumps for each 32nd difference. Figure 11.3 illustrates the spread between the December and September futures contract. At the time this graph was produced, short-term interest rates were lower than the yield on U.S. Treasury bonds; therefore, U.S. Treasury bond futures contracts exhibited negative carry and hence the spread was negative. Figure 11.4 illustrates short-term interest rates for this same period, indicating that interest rates are an important component of carrying costs. There appears a general relationship between short-term interest rates and Treasury bond futures spreads.

FIGURE 11.3 Changes in Spread U.S. Treasury Bond Futures Contracts

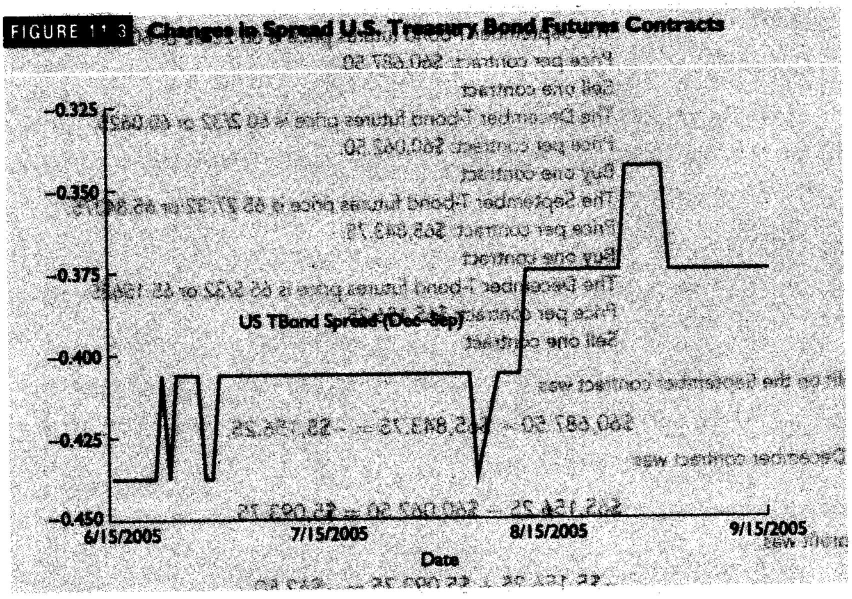


FIGURE 11.4 Short-Term Interest Rates

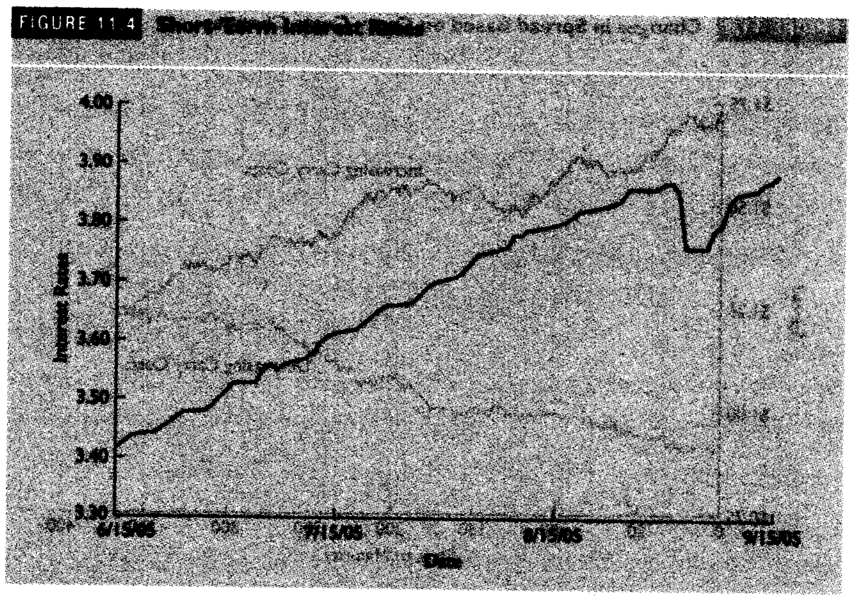


Table 11.12 presents an example of a simple spread taken in anticipation of changes in interest rates in the T-bond market. The trader goes short the September contract and long the December in the hope

Table 11.12 A Treasury Bond Futures Spread

Scenario: It is July 6. Interest rates have steadily risen over the last six months. The yield on long-term government bonds is 13.54 percent. A Treasury bond futures floor trader anticipates that rates will continue upward; however, the economy remains healthy and there are no indications that the Fed will tighten the money supply, which would drive rates further upward. Thus, while the trader is bearish she is encouraged by other economic factors. She wants to take a speculative short position in T-bond futures but is concerned that rates will fall, generating a potentially large loss. She believes that if rates have not changed by late August, they will not change at all. Thus, she shorts the September contract and buys the December contract.

Date	Futures Market
July 6	The September T-bond futures price is 60 22/32 or 60.6875. Price per contract: $60,687.50. **Sell one contract** The December T-bond futures price is 60 2/32 or 60.0625. Price per contract: $60,062.50. **Buy one contract**
August 31	The September T-bond futures price is 65 27/32 or 65.84375. Price per contract: $65,843.75. **Buy one contract** The December T-bond futures price is 65 5/32 or 65.15625. Price per contract: $65,156.25. **Sell one contract**

Analysis: The profit on the September contract was

$$\$60{,}687.50 - \$65{,}843.75 = -\$5{,}156.25.$$

The profit on the December contract was

$$\$65{,}156.25 - \$60{,}062.50 = \$5{,}093.75.$$

Thus, the overall profit was

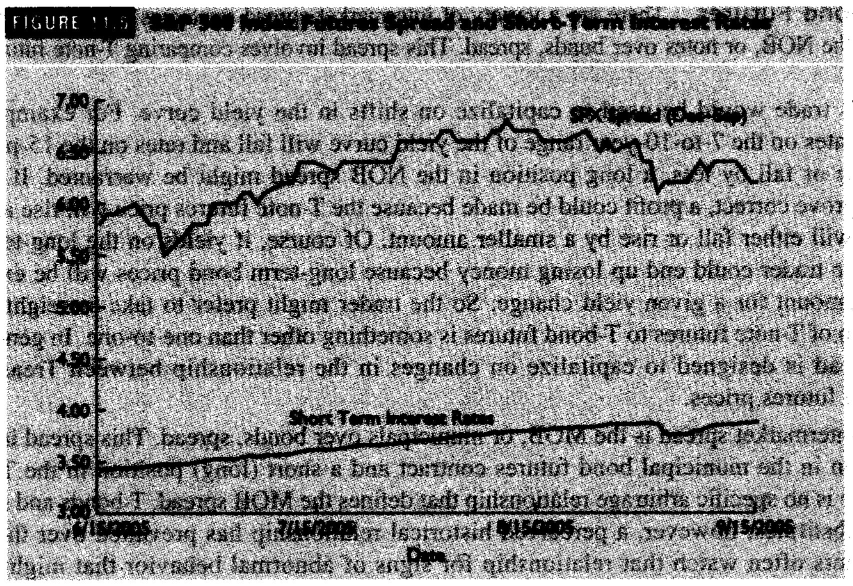
$$-\$5{,}156.25 + \$5{,}093.75 = -\$62.50.$$

that a small gain will be made. If the trader's forecast is wrong, the loss is not likely to be large. This example involves a simple position of one contract long and one contract short. In practice many traders use weighted positions, taking more of either the long or short contract so as to balance their different volatilities. The proper number of each contract to provide a risk-free position is obtained by using their modified durations in the same manner that we did above when we constructed long spot and short futures positions. In this example, the trader's expectations were not at all realized. Rates fell and bond futures prices rose. The September contract was not profitable, but the long December contract was profitable and the overall loss was negligible.

Stock Index Futures Spreads The factors determining the carry costs for stock index futures contracts are interest rates and dividends. Dividends are usually more predictable than interest rates, although somewhat tedious; hence, the driving force determining changes in spreads should be interest rates.

Figure 11.5 illustrates the intramarket spread for the S&P 500 index futures contracts of September and December. A rough relationship clearly exists between short-term interest rates and equity futures spreads.

FIGURE 11.5 S&P 500 Index Futures Spread and Short-Term Interest Rates



Intermarket Spreads

In the previous section we discussed three forms of intramarket spreads. There are also a number of intermarket spreads or intercommodity spreads. These are transactions in which the two futures contracts are on different underlying instruments. There are a vast number of different intermarket spreads. We first review the generic futures case, highlighting the difference from intramarket spread strategies and then only briefly review bond and stock strategies.

Generic Intermarket Futures Spreads Intermarket spreads tend to be more risky because the two positions do not share a common underlying instrument. Thus, intermarket spreads not only change in value due to changes in the carry costs but also change due to changes in the value of the underlying instrument as can be seen in the following identity:

$$f_0(C2) - f_0(C1) = S_{0,C2} + \theta_{0,C2} - [S_{0,C1} + \theta_{0,C1}].$$

This equation defines the spread between futures prices involving two different underlying instruments (denoted C1 and C2), where we assume you are long the C2 futures contract and short the C1 futures contract. Recall that S_0 denotes the underlying price at time 0, and θ_0 denotes the carrying costs at time 0. Therefore, the profit from a spread trade can be expressed as the profit on the two positions,

$$\begin{aligned}\Pi &= [f_t(C2) - f_0(C2)] - [f_t(C1) - f_0(C1)] \\ &= [S_{t,C2} + \theta_{t,C2} - S_{0,C2} - \theta_{0,C2}] - [S_{t,C1} + \theta_{t,C1} - S_{0,C1} - \theta_{0,C1}] \\ &= \{[S_{t,C2} + S_{t,C1}] - [S_{0,C2} - S_{0,C1}]\} + \{[\theta_{t,C2} - \theta_{t,C1}] - [\theta_{0,C2} - \theta_{0,C1}]\},\end{aligned}$$

which is two sets of differences, one being the difference between the change in price of the underlying instrument of contracts 2 and 1 and the other being the difference between the change in the carry costs of contract 2 and 1. Therefore, the predictions are more complex. Often the volatility of changes in the underlying instruments is much higher than the volatility of carry costs. Hence, the spread trader's main task relates to predicting changes in the prices and carrying costs of the underlying instruments.

Note and Bond Futures There are a variety of intermarket spread strategies within the debt market. We examine here the NOB, or notes over bonds, spread. This spread involves comparing T-note futures and T-bond futures.

The NOB trade would be used to capitalize on shifts in the yield curve. For example, if a trader believes that rates on the 7-to-10-year range of the yield curve will fall and rates on the 15-plus-year range will either rise or fall by less, a long position in the NOB spread might be warranted. If the investor's expectations prove correct, a profit could be made because the T-note futures price will rise and the T-bond futures price will either fall or rise by a smaller amount. Of course, if yields on the long-term end of the market fall, the trader could end up losing money because long-term bond prices will be expected to rise by a greater amount for a given yield change. So the trader might prefer to take a weighted position in which the ratio of T-note futures to T-bond futures is something other than one-to-one. In general, however, the NOB spread is designed to capitalize on changes in the relationship between Treasury note and Treasury bond futures prices.

Another intermarket spread is the MOB, or municipals over bonds, spread. This spread involves a long (short) position in the municipal bond futures contract and a short (long) position in the T-bond futures contract. There is no specific arbitrage relationship that defines the MOB spread. T-bonds and municipals are not perfect substitutes; however, a perceived historical relationship has prevailed over the past several decades. Traders often watch that relationship for signs of abnormal behavior that might signal profit opportunities.

Stock Index Futures Intermarket spread trading can also be performed with stock index futures contracts. Spread traders may have a view on the relative performance of one equity sector when compared to another. With the addition of single stock futures contracts, spread trading with futures contracts can now involve two individual stocks or one individual stock and an equity index. As with other intermarket spread trading activities, the success of the spread trader is directly related to forecasting skill.

TARGET STRATEGIES

We now turn to various target strategies. Specifically we explore target duration strategies with bond futures, alpha capture, target beta strategies with stock index futures, and tactical asset allocation. There are many other possible target strategies to consider, but the ones covered here will give an adequate overview of target strategies.

Target Duration with Bond Futures

We showed how hedge ratios can be constructed using futures. The procedure essentially combines futures so that the overall investment is insensitive to interest rate changes. In effect, this changes the modified duration to zero. Suppose a market timer believes that interest rates will move in one direction or the other, but the timer does not want to completely eliminate the exposure by taking the modified duration to zero. If interest rates are expected to fall, the timer may wish to increase the modified duration; if rates are expected to rise, the timer may wish to decrease the modified duration but not necessarily reduce it to zero.

Suppose a portfolio of bonds has a market value of B and a modified duration of MD_B. The futures contract has a modified duration of MD_f and a price of f. The timer wishes to increase the spot modified duration to MD_T, which we shall call the target duration. One way to do this is to put more money in high modified duration bonds and less money in low modified duration bonds, but this would incur transaction costs on the purchase and sale of at least two bonds. Futures can be used to easily adjust the modified duration. Also, by doing the transaction with futures rather than buying and selling the spot instruments, there are significant savings in transaction costs.

The number of futures needed to change the modified duration to MD_T is[15]

$$N_f = \left(\frac{MD_T - MD_B}{MD_f}\right)\left(\frac{B}{f}\right).$$

Notice how similar this formula is to the one given earlier in this chapter. That formula was

$$N_f^* = \left(\frac{MD_B}{MD_f}\right)\left(\frac{B}{f}\right),$$

which reduces the interest sensitivity and modified duration to zero.[16] N_f^* denotes the optimal hedge ratio where the objective is to minimize price volatility. Here we will not use the asterisk,*, when seeking to change modified duration to a non-zero target. If the target duration were zero in our new formula, we would obtain the old formula. Thus, the new formula is a much more general restatement of the optimal hedge ratio covered earlier in this chapter, because it permits the modified duration to be adjusted to any chosen value. If the investor expected falling interest rates and wished to increase the modified duration, MD_T would be larger than MD_B. Then N_f would be positive and futures would be bought. This makes sense because adding futures to a long spot position should increase the risk. If the trader is bearish and wishes to reduce the modified duration, MD_T would be less than MD_B and N_f would be negative, meaning that futures would be sold. This is so because an opposite position in futures should be required to reduce the risk.

An Example Let us rework the T-bond hedging example that appeared in Table 11.7. In that problem, on February 25 the portfolio manager held \$1 million face value of the 11 7/8s bond maturing in about 25 years. The bond price is 101, and the bond will be sold on March 28. In that example, we feared an increase in interest rates and lowered the modified duration to zero by selling 16 futures contracts. Suppose, however, that we wanted to lower the modified duration from its present level of 7.83 to 4. This would make the portfolio less sensitive to interest rates, but not completely unaffected by them. In that way, if the forecast proved incorrect, the positive modified duration would still leave room to profit.

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[15]See Hedge Ratios and Futures Contracts Technical Note in the section called "Derivation of Price Sensitivity Hedge Ratio."

[16]As noted above, both of these formulas apply only to extremely short holding periods. Any increment of time or change in yield will require recalculation of the number of futures contracts, so the number of contracts will need adjustment throughout the holding period.

The example is presented in Table 11.13. The bond ends up losing 2.26 percent. To determine how close the outcome was to the desired outcome, we need to know the change in the yield on the bonds. On February 25, the yield on the spot bond was 11.74 percent; on March 28, the yield that corresponded to a price of 95 22/32 was 12.50 percent. Recall that the following formula expresses the relationship between the change in the yield and the percentage change in the bond price:

$$\left(\frac{\Delta B}{B}\right) \approx -MD(\Delta y),$$

Table 11.13 Target Duration with Bond Futures

Scenario: On February 25, a portfolio manager holds $1 million face value of a government bond, the 11 7/8s, which matures in about 25 years. The bond is currently priced at 101, has a modified duration of 7.83, and has a yield of 11.74 percent. The manager plans to sell the bond on March 28. The manager is worried about rising interest rates and would like to reduce the bond's sensitivity to interest rates by lowering its modified duration to 4. This would reduce its interest sensitivity, which would help if rates increase, but would not eliminate the possibility of gains from falling rates.

Date	Spot Market	Futures Market
February 25	The current price of the bonds is 101. Value of position: $1,010,000. The short end of the term structure is flat, so this is the forward sale price of the bonds in March.	June T-bond futures is at 70 16/32. Price per contract: $70,500. The futures price and the characteristics of the deliverable bond imply a modified duration of 7.20 and a yield of 14.92 percent. Appropriate number of contracts: $N_f = -\left(\frac{4-7.83}{7.20}\right)\left(\frac{1,010,000}{70,500}\right) = -7.62.$ **Sell 8 contracts**
March 28	The bonds are sold at their current price of 95 22/32. This is a price of $956.875 per bond. Value of position: $956,875.	June T-bond futures is at 66 22/32. Price per contract: $66,718.75. **Buy 8 contracts**

Analysis: When the $1 million face value bonds are sold on March 28, they are worth $956,875, a loss in value of $1,010,000 – $956,875 = $53,125.

The profit on the futures transaction is

8($70,500) (sale price of futures)
<u>–8($66,718.75)</u> (purchase price of futures)
$30,250 (profit on futures).

Thus, the overall transaction resulted in a loss in value of $53,125 – $30,250 = $22,875, which is a 2.26 percent decline in overall value.

where B represents the bond price. With a modified duration reset to 4, the formula predicts that for a yield change of 0.1250 – 0.1174 = 0.0076, the percentage price change would be

$$\frac{\Delta B}{B} \approx -4(0.0076) = -0.0304,$$

or 3.04 percent. The actual change was 2.26 percent. Without the futures position, the modified duration still would have been 7.83. Plugging into the formula gives a predicted percentage change in the bond price of

$$\frac{\Delta B}{B} \approx -7.83\,(0.0076) = -0.0595,$$

or a loss of 5.95 percent. The actual change without the hedge would have been −\$53,125/\$1,010,000 = −0.0526, or a 5.26 percent loss.

Alpha Capture

Recall that the systematic risk of a diversified portfolio can be hedged by using stock index futures. Because the portfolio is diversified, there is no unsystematic risk and therefore the portfolio is riskless. In some cases, however, an investor might wish to hedge the systematic risk and retain the unsystematic risk.

The unsystematic return that reflects investment performance over and above the risk associated with the systematic or market-wide factor is called *alpha*. In an efficient market, investors cannot expect to earn positive alpha. Professional financial analysts, however, do not believe this is true. Thousands of analysts devote all of their time to identifying overpriced and underpriced stocks. An analyst who thinks a stock is underpriced normally recommends it for purchase. If the stock is purchased and the market goes down, the stock's overall performance can be hurt. For example, a drug firm might announce an important new drug that can help cure diabetes. If that announcement occurs during a strong bear market, the stock can be pulled down by the market effect. The extent of the stock's movement with the market is measured by its beta. In this section we shall look at how alpha can be captured by using stock index futures to eliminate the systematic component of performance, leaving the unsystematic performance, the alpha. In this sense, the strategy is based on a target beta of zero.

We will use the following notation:

$S =$ stock price
$M =$ value of market portfolio of all risky assets
$r_S = \Delta S/S =$ rate of return on stock
$r_M = \Delta M/M =$ rate of return on market
$\beta =$ beta of the stock

The stock's return consists of its systematic return, βr_M, and its unsystematic return or alpha, denoted as α. Thus,

$$r_S = \beta r_M + \alpha.$$

If we multiply both sides of the equation by S, we have

$$Sr_S = S\beta r_M + S\alpha,$$

which is equivalent to

$$\Delta S = S\beta(\Delta M/M) + S\alpha.$$

This is the return on the stock expressed in dollars. The objective of the transaction is to capture a profit equal to the dollar alpha, $S\alpha$.

The profit from a transaction consisting of the stock and N_f futures contracts is

$$\Pi = \Delta S + N_f^* \Delta f.$$

Recall that the formula for in an ordinary stock index futures hedge is $-\beta(S/f)$. Let us substitute this for N_f^*:

$$\Pi = \Delta S - \beta \left(\frac{S}{f}\right) \Delta f.$$

Now we need to substitute $S\beta(\Delta M/M) + S\alpha$ for ΔS and assume the futures price change will match the index price change. In that case, $\Delta M/M = \Delta f/f$. Making these substitutions gives

$$\Pi = S\alpha.$$

Thus, if we use N_f^* futures contracts where N_f^* is the ordinary hedge ratio for stock index futures, we will eliminate systematic risk and the profit will be the dollar alpha.

An Example Table 11.14 illustrates an application of this result. In this example, you have identified what you believe to be an underpriced stock. You are worried, however, that the market as a whole will decline and drag the stock down. To hedge the market effect, you sell stock index futures.

Table 11.14 Alpha Capture

Scenario: On July 1, you are following the stock of Helene Curtis, which has a price of 17.38 and a beta of 1.10. Barring any change in the general level of stock prices, you expect the stock to appreciate by about 10 percent by the end of September. Your analysis of the market as a whole calls for about an 8 percent decline in stock prices in general over the same period. Since the stock has a beta of 1.10, this would bring the stock down by 1.10(.08) = 0.088, or 8.8 percent, which will almost completely offset the expected 10 percent unsystematic increase in the stock price. You decide to hedge the market effect by selling stock index futures.

Date	Spot Market	Futures Market
July 1	Own 150,000 shares of Helene Curtis stock at 17.38. Value of stock: 150,000($17.38) = $2,607,000.	December stock index futures price is 444.60. Price of one contract: 444.60($500) = $222,300. Appropriate number of contracts: $N_f = 1.10\left(\frac{\$2,607,000}{\$222,300}\right) = -12.90.$ **Sell 13 contracts**
September 30	Stock price is 17.75. Value of stock; 150,000($17.75) = $2,662,500.	December stock index futures price is 411.30. Price of one contract: 411.30($500) = $205,650. **Buy 13 contracts**

Analysis: Profit on stock:

$$\begin{array}{r} \$2,662,500 \\ \underline{-\$2,607,000} \\ \$55,500. \end{array}$$

Profit on futures:

$$\begin{array}{r} 13(\$222,300) \\ \underline{-13(\$205,650)} \\ \$216,450. \end{array}$$

Overall profit:

$$\begin{array}{r} \$55,500 \\ \underline{+\$216,450} \\ \$271,950. \end{array}$$

Overall rate of return:

$$\frac{\$271,950}{\$2,607,000} = 0.1043.$$

This transaction should not be considered riskless. Suppose your analysis is incorrect and the company announces some bad news during a bull market. Selling the futures contract would eliminate the effect of the bull market while retaining the effect of the bad news announcement. Moreover, this type of trade depends not only on the correctness of the analysis but on the beta's stability.

Target Beta with Stock Index Futures

Recall in the hedging discussion above we considered a portfolio manager who sold stock index futures to eliminate the systematic risk. At a later date, the futures contracts were repurchased and the portfolio was returned to its previous level of systematic risk. Assuming the beta of the futures is 1, the number of futures contracts was given by the formula

$$N_f^* = -\beta_S \frac{S}{f}.$$

In some cases, a portfolio manager may wish to change the systematic risk but not eliminate it altogether. For example, if a portfolio manager believes the market is highly volatile, the portfolio beta could be lowered but not reduced to zero. This would enable the portfolio to profit if the market did move upward but would produce a smaller loss if the market moved down. At more optimistic times, the portfolio beta could be increased. In the absence of stock index futures (or options), changing the portfolio beta would require costly transactions in the individual stocks.

Assume we have a portfolio containing stock valued at S and N_f futures contracts. We drop the asterisk, *, because we are not focused on the minimum risk solution now. The return on the portfolio is given as r_{Sf}, where

$$r_{Sf} = \frac{\Delta S + N_f \Delta f}{S}.$$

The first term in the numerator, ΔS, is the change in the price of the stock. The second term, $N_f \Delta f$, is the number of contracts times the change in the price of the futures contract. The denominator, S, is the amount of money invested in the stock. The expected return on the portfolio, $E(r_{Sf})$, is

$$E(r_{Sf}) = \frac{E(\Delta S)}{S} + N_f \frac{E(\Delta f)}{S} = E(r_s) + \frac{N_f}{S} E(\Delta f).$$

where $E(r_S)$ is the expected return on the stock defined as $E(\Delta S)/S$ and $E(\Delta f)$ is the expected change in the price of the futures contract.

From modern portfolio theory, the Capital Asset Pricing Model (CAPM) gives the expected return on a stock as $r + [E(r_M) - r]\beta$, where the beta is the systematic risk and reflects the influence of the market as a whole on the stock. If the market is efficient, the investor's required return will equal the expected return. If the CAPM holds for stocks, it should also hold for stock index futures; however, it would be written as

$$\frac{E(\Delta f)}{f} = [E(r_M) - r]\beta_f = E(r_M) - r,$$

where β_f is the beta of the futures contract, assumed to be 1. Although in reality β_f will not be precisely equal to 1, it is sufficiently close that we shall assume it to keep the example simple. Note that this CAPM equation seems to be missing the term r from the right-hand side. The risk-free rate reflects the opportunity cost of money invested in the asset. Because the futures contract requires no initial outlay, there is no opportunity cost; thus, the r term is omitted. Thus, a long position in a stock index futures contract would be expected to return the market risk premium and a short position in a stock index futures contract would be expected to lose the market risk premium.

The objective is to adjust the portfolio beta, $_S$, and expected return, $E(r_{Sf})$, to a more preferred level. Since the CAPM holds for the portfolio, we can write the relationship between expected return and beta as

$$E(r_{Sf}) = r + [E(r_M) - r]\beta_T,$$

where β_T is the target beta, the desired risk level. Now we substitute for $E(r_S)$ and $E(\Delta f)$ and get

$$E(r_{Sf}) = r + [E(r_M) - r]\beta_S + N_f(f/S)[E(r_M) - r].$$

Setting this equal to $r + [E(r_M) - r]\beta_T$ and solving for N_f gives

$$N_f = (\beta_T - \beta_s)\left(\frac{S}{f}\right)$$

This formula differs only slightly from the previous formula of N_f^*, which was covered in our hedging material earlier in this chapter. In fact, the N_f^* formula is but a special case of this one. For example, if the target beta is zero, the above formula reduces to $-\beta_S\left(\frac{S}{f}\right)$, where the negative sign means that you would sell N_f futures. This is the same formula we used in the hedging discussion above to eliminate systematic risk.

When the manager wants to increase the beta, β_T will be greater than β_S and N_f will be positive. In that case, the manager will buy futures contracts. That makes sense, since the risk will increase. When the beta needs to be reduced, β_T will be less than β_S and the manager should sell futures to reduce the risk.

An Example Table 11.15 presents an example in which the manager of a portfolio that has a beta of 0.95 would like to temporarily increase the beta to 1.25. The manager buys stock index futures, which results in an overall gain of almost 12 percent when the stocks themselves gained only 10 percent.

Tactical Asset Allocation Using Stock and Bond Futures

A typical investment portfolio often consists of money allocated to certain classes of assets. Stock is one general class while bonds are another. Some portfolios would contain additional asset classes such as real estate, hedge funds, venture capital, or commodities. Within a given asset class, there may be subdivisions into more specific classes. For example, the stock asset class could be divided into domestic stock and international stock, or perhaps large-cap stock, mid-cap stock, and small-cap stock.[17] Portfolio managers then usually try to adjust the allocation of funds among the asset classes in such a manner that the most attractive classes receive the highest weights. This form of portfolio management is usually called *asset allocation*.

Over the long run, a portfolio typically has a specified set of target weights for each asset class. These weights are usually called the *strategic asset allocation*. In the short run, the portfolio manager can deviate from these weights by allocating more funds to the classes expected to provide the best performance. These short run deviations from the strategic asset allocation make up a process called *tactical asset allocation*. Of course, such a strategy is just a variation of target strategies, as we have discussed with respect to both stock and bond futures.

The execution of tactical asset allocation strategies can be done by buying and selling assets within the various classes. When futures are available on underlyings that are sufficiently similar to the asset classes, they are often used to execute these strategies in a more efficient, less costly manner. Let us see how this can be done.

[17]Cap is an abbreviation for market capitalization, or the current market value of outstanding common stock.

Table 11.15 Target Beta Strategies with Stock Index Futures

Scenario: On August 29, a portfolio manager is holding a portfolio of stocks worth $3,783,210. The portfolio beta is 0.95. The manager expects the stock market as a whole to appreciate substantially over the next three months and wants to increase the portfolio beta to 1.25. The manager could buy and sell shares in the portfolio, but this would incur high transaction costs and later the portfolio beta would have to be adjusted back to 0.95. The manager decides to buy stock index futures to temporarily increase the portfolio's systematic risk. The prices, number of shares, and betas are given below. The target date for evaluating the portfolio is November 29.

Stock	Price (8/29)	Number of Shares	Market Value	Weight	Beta
Beneficial Corp.	40.50	11,350	$459,675	0.122	0.95
Cummins Engine	64.50	10,950	706,275	0.187	1.10
Gillette	62.00	12,400	768,800	0.203	0.85
Kmart	33.00	5,500	181,500	0.048	1.15
Boeing	49.00	4,600	225,400	0.059	1.15
W. R. Grace	42.62	6,750	287,685	0.076	1.00
Eli Lilly	87.38	11,400	996,132	0.263	0.85
Parker Pen	20.62	7,650	157,743	0.042	0.75
			$3,783,210	1.000	

Portfolio beta:

$$0.122(0.95) + 0.187(1.10) + 0.203(0.85) + 0.048(1.15) + 0.059(1.15) + 0.076(1.00) + 0.263(0.85) + 0.042(0.75) = 0.95.$$

December stock index futures contract:

Price on August 29: 759.60
Multiplier: $250
Price of one contract: $250(759.60) = $189,900.

Required number of futures contracts:

$$N_f = (1.25 - 0.95)\left(\frac{3,783,210}{189,900}\right) = 5.98.$$

Buy 6 contracts

Results: The values of the stocks on November 29 are shown below:

Stock	Price (11/29)	Market Value
Beneficial Corp.	45.13	$512,226
Cummins Engine	66.75	730,913
Gillette	69.87	866,388
Kmart	35.12	193,160
Boeing	49.12	225,952
W. R. Grace	40.75	275,062
Eli Lilly	103.75	1,182,750
Parker Pen	22.88	175,032
		$4,161,483

December stock index futures contract:

Price on August 29: 809.60
Multiplier: $250
Price of one contract: $250(809.60) = $202,400.

Sell 6 contracts

Analysis: The market value of the stocks increased by $4,161,483 – $3,783,210 = $378,273, a gain of about 10 percent. The futures profit was

6($202,400)	(sale price of futures)
–6($189,900)	(purchase price of futures)
$75,000	(profit on futures).

Thus, the overall gain on the portfolio was effectively increased to $378,273 + $75,000 = $453,273, a return of about 12 percent.

Consider a portfolio with just two asset classes, stock and bonds. The stock asset class has a specific beta, and the bond asset class has a specific modified duration. If the manager wants to decrease the allocation to stock and increase the allocation to bonds, she can do so by selling stocks and buying bonds. Alternatively, she may be able to sell stock futures and buy bond futures to achieve the desired result. Of course, whether futures are an acceptable substitute for transactions in the actual securities depends on whether the stock index underlying the stock index futures is similar enough to the stock component of her portfolio and whether the bond underlying the bond futures is similar enough to the bond component of her portfolio. Since practitioners use these instruments so often to make asset allocation changes, we can safely assume that these conditions hold so that stock and bond futures can be used for this purpose.

Suppose that she wishes to sell a certain amount of stock and buy an equivalent amount of bonds. Using the formula we previously showed for target beta strategies with stock index futures, she will sell the number of futures contracts to adjust the beta on that given amount of stock from its current level to zero. This transaction has the effect of selling the stock and converting it to cash. Now she wants to convert this synthetically created cash to bonds. She will then buy bond futures to adjust the modified duration on this synthetic cash from zero to its desired level, which is the modified duration of the existing bond component of the portfolio.

Note that the securities in the portfolio have not changed, but the allocation between stock and bonds has been synthetically altered by the addition of a short position in stock index futures and a long position in bond futures. Now suppose she also would like to change the risk characteristics of the existing stock and bond asset classes. She can then adjust the beta on the stock by buying or selling more stock index futures contracts, and she can adjust the modified duration on the bonds by buying or selling more bond futures contracts.

An Example Table 11.16 illustrates this type of strategy. Here the portfolio manager wants to reduce the allocation from two-thirds stock, one-third bonds to half stock, half bonds. In addition, the manager wants to lower the risk exposure on the stock and raise the risk exposure on the bonds.

Table 11.16 Tactical Asset Allocation Using Stock and Bond Futures

Scenario: A \$30 million portfolio consists of \$20 million of stock at a beta of 1.15 and \$10 million of bonds at a modified duration of 6.25 with a yield of 7.15 percent. The manager would like to change the allocation to \$15 million of stock and \$15 million of bonds. In addition, the manager would like to adjust the beta on the stock to 1.05 and the modified duration on the bonds to 7. A stock index futures contract has a price of \$225,000, and we can assume that its beta is 1.0. A bond futures contract is priced at \$92,000 with an implied modified duration of 5.9 and an implied yield of 5.65 percent. The manager will use futures to synthetically sell \$5 million of stock, reduce the beta on the remaining stock, synthetically buy \$5 million of bonds, and increase the modified duration on the remaining bonds. The horizon date is three months.

Step 1. Synthetically sell \$5 million of stock

This transaction will effectively reduce the beta on \$5 million of stock to zero, thereby synthetically converting the stock to cash. The number of stock futures, which we denote as N_{sf}, will be

$$N_{sf} = (0 - 1.15)\left(\frac{5{,}000{,}000}{225{,}000}\right) = -25.56.$$

This rounds off to selling 26 contracts. After executing this transaction, the portfolio effectively consists of \$15 million of stock at a beta of 1.15, \$10 million of bonds at a modified duration of 6.25, and \$5 million of synthetic cash. Of course, the actual portfolio consists of \$20 million of stock at a beta of 1.15, \$10 million of bonds at a modified duration of 6.25, and \$5 million of short stock index futures.

Step 2. Synthetically buy \$5 million of bonds

This transaction will effectively convert the \$5 million of synthetic cash, which can be treated as a bond with a modified duration of zero, to \$5 million of synthetic bonds with a modified duration of 6.25. The number of bond futures, which we denote as N_{bf}, will be

Table 11.16 continued

$$N_f = \left(\frac{6.25 - 0}{5.9}\right)\left(\frac{5{,}000{,}000}{92{,}000}\right) = 56.6.$$

This rounds off to buying 57 contracts. After executing this transaction, the portfolio effectively consists of $15 million of stock at a beta of 1.15 and $15 million of bonds at a modified duration of 6.25. Of course, the actual portfolio consists of $20 million of stock at a beta of 1.15, $10 million of bonds at a modified duration of 6.25, $5 million of short stock index futures, and $5 million of long bond futures.

Step 3. Lower the beta on the stock from 1.15 to 1.05

Now the manager wants to lower the beta from 1.15 to 1.05 on $15 million of stock. This will require

$$N_{sf} = (1.05 - 1.15)\left(\frac{15{,}000{,}000}{225{,}000}\right) = -6.67.$$

contracts. Rounding off, the manager would sell 7 contracts. In the aggregate, the manager would sell 33 stock index futures contracts.

Step 4. Raise the modified duration on the bonds from 6.25 to 7

Now the manager wants to raise the modified duration from 6.25 to 7 on $15 million of bonds. This will require

$$N_{bf} = \left(\frac{7 - 6.25}{5.9}\right)\left(\frac{15{,}000{,}000}{92{,}000}\right) = 20.73.$$

contracts. Rounding off, the manager would buy 21 contracts. In the aggregate, the manager would buy 78 bond futures contracts.

It is important to know that the same results would be obtained if the transactions were carried out in a different order. The manager could first reduce the beta on the $20 million of stock and then synthetically sell $5 million of stock. The manager could then increase the modified duration on the $10 million of bonds and then synthetically buy $5 million of bonds.

Results

Three months later, the stock is worth $19,300,000, and the bonds are worth $10,100,000. The stock index futures price falls to $217,800, and the bond futures price rises to $92,878. The profit on the stock index futures transaction is

$$-33(\$217{,}800 - \$225{,}000) = \$237{,}600.$$

The profit on the bond futures transaction is

$$78(\$92{,}878 - \$92{,}000) = \$68{,}484.$$

The overall value of the portfolio is

Stock	$19,300,000
Stock index futures profit	237,600
Bonds	10,100,000
Bond futures profit	68,484
Total	$29,706,084.

Had the transactions not been executed, the portfolio would have been worth

Stock	$19,300,000
Bonds	10,100,000
Total	$29,400,000.

Of course, this transaction is speculative. The reduction of the stock allocation and reduction of the stock beta, combined with the increase in the bond allocation and the increase in the bond beta, was a good move, but it could have been a bad one. In any case, however, derivatives allowed these transactions to be executed synthetically and less expensively.

QUESTIONS AND PROBLEMS

1. On June 17 of a particular year, an American watch dealer decided to import 100,000 Swiss watches. Each watch costs SF225. The dealer would like to hedge against a change in the dollar/Swiss franc exchange rate. The forward rate was $0.3881. Determine the outcome from the hedge if it was closed on August 16, when the spot rate was $0.4434.
2. On January 31, a firm learns that it will have $5 million available on May 31. It will use the funds to purchase the APCO 9 1/2 percent bonds maturing in about 21 years. Interest is paid semiannually on March 1 and September 1. The bonds are rated A2 by Moody's and are selling for 78 7/8 per 100 and yielding 12.32 percent. The modified duration is 7.81.
   The firm is considering hedging the anticipated purchase with September T-bond futures. The futures price is 71 8/32. The firm believes the futures contract is tracking the Treasury bond with a coupon of 12 3/4 percent and maturing in about 25 years. It has determined that the implied yield on the futures contract is 11.40 percent and the modified duration of the contract is 8.32. The firm believes the APCO bond yield will change 1 point for every 1-point change in the yield on the bond underlying the futures contract.
   a. Determine the transaction the firm should conduct on January 31 to set up the hedge.
   b. On May 31, the APCO bonds were priced at 82 3/4. The September futures price was 76 14/32. Determine the outcome of the hedge.
3. For each of the following hedge termination dates, identify the appropriate contract expiration. Assume the available expiration months are March, June, September, and December.
   a. August 10
   b. December 15
   c. February 20
   d. June 14
4. For each of the following situations, determine whether a long or short hedge is appropriate. Justify your answers.
   a. A firm anticipates issuing stock in three months.
   b. An investor plans to buy a bond in 30 days.
   c. A firm plans to sell some foreign currency denominated assets and convert the proceeds to domestic currency.
5. Explain how to determine whether to buy or sell futures when hedging.
6. Explain the difference between a short hedge and a long hedge.
7. You are the manager of a stock portfolio. On October 1, your holdings consist of the eight stocks listed in the following table, which you intend to sell on December 31. You are concerned about a market decline over the next three months. The number of shares, their prices, and the betas are shown, as well as the prices on December 31.

Stock	Number of Shares	Beta	10/1 Price	12/31 Price
R. R. Donnelley	10,000	1.00	19.63	27.38
B. F. Goodrich	6,200	1.05	31.38	32.88
Raytheon	15,800	1.15	49.38	53.63
Maytag	8,900	0.90	55.38	77.88
Kroger	11,000	0.85	42.13	47.88
Comdisco	14,500	1.45	19.38	28.63
Cessna	9,900	1.20	29.75	30.13
Foxboro	4,500	0.95	24.75	26.00

On October 1, you decide to execute a hedge using a stock index futures contract, which has a $500 multiplier. The March contract price is 376.20. On December 31, the March contract price is 424.90. Determine the outcome of the hedge.

8. On July 1, a portfolio manager holds $1 million face value of Treasury bonds, the 11 1/4s maturing in about 29 years. The price is 107 14/32. The bond will need to be sold on August 30. The manager is concerned about rising interest rates and believes a hedge would be appropriate. The September T-bond futures price is 77 15/32. The price sensitivity hedge ratio suggests that the firm should use 13 contracts.
   a. What transaction should the firm make on July 1?
   b. On August 30, the bond was selling for 101 12/32 and the futures price was 77 5/32. Determine the outcome of the hedge.
9. On March 1, a securities analyst recommended General Cinema stock as a good purchase in the early summer. The portfolio manager plans to buy 20,000 shares of the stock on June 1 but is concerned that the market as a whole will be bullish over the next three months. General Cinema's stock currently is at 32.88, and the beta is 1.10.
   Construct a hedge that will protect against movements in the stock market as a whole. Use the September stock index futures, which is priced at 375.30 on March 1 and which has a $500 multiplier. Evaluate the outcome of the hedge if on June 1 the futures price is 387.30 and General Cinema's stock price is 38.63.
10. During the first six months of the year, yields on long-term government debt have fallen about 100 basis points. You believe the decline in rates is over, and you are interested in speculating on a rise in rates. You are, however, unwilling to assume much risk, so you decide to do an intramarket spread. Use the following information to construct a T-bond futures spread on July 15, and determine the profit when the position is closed on November 15.

July 15
December futures price: 76 9/32
March futures price: 75 9/32

November 15
December futures price: 79 13/32
March futures price: 78 9/32

11. The manager of a $20 million portfolio of domestic stocks with a beta of 1.10 would like to begin diversifying internationally. He would like to sell $5 million of domestic stock and purchase $5 million of foreign stock. He learns that he can do this using a futures contract on a foreign stock index. The index is denominated in dollars, thereby eliminating any currency risk. He would like the beta of the new foreign asset class to be 1.05. The domestic stock index futures contract is priced at $250,000 and can be assumed to have a beta of 1.0. The foreign stock index futures contract is priced at $150,000 and can also be assumed to have a beta of 1.0.
   a. Determine the number of contracts he would need to trade of each type of futures in order to achieve this objective.
   b. Determine the value of the portfolio if the domestic stock increases by 2 percent, the domestic stock futures contract increases by 1.8 percent, the foreign stock increases by 1.2 percent, and the foreign stock futures contract increases by 1.4 percent.
12. On November 1, an analyst who has been studying a firm called Computer Sciences believes the company will make a major new announcement before the end of the year. Computer Sciences currently is priced at 27.63 and has a beta of 0.95. The analyst believes the stock can advance about 10 percent if the market does not move. The analyst thinks the market might decline by as much as 5 percent, leaving the stock with a return of $0.10 + (-0.05)(0.95) = 0.0525$. To capture the full 10 percent alpha, the analyst recommends the sale of stock index futures. The March contract currently is priced at 393. Assume the investor owns 100,000 shares of the stock. Set up a transaction by determining the appropriate number of futures contracts. Then

determine the effective return on the stock if, on December 31, the stock is sold at 28.88, the futures contract is at 432.30, and the multiplier is 500. Explain your results.

13. You are the manager of a stock portfolio worth $10,500,000. It has a beta of 1.15. During the next three months, you expect a correction in the market that will take the market down about 5 percent; thus, your portfolio is expected to fall about 5.75 percent (5 percent times a beta of 1.15). You wish to lower the beta to 1. A stock index futures contract with the appropriate expiration is priced at 425.75 with a multiplier of $500.
    a. Should you buy or sell futures? How many contracts should you use?
    b. In three months, the portfolio has fallen in value to $9,870,000. The futures has fallen to 402.35. Determine the profit and portfolio return over the quarter. How close did you come to the desired result?
14. On January 2 of a particular year, an American firm decided to close out its account at a Canadian bank on February 28. The firm is expected to have 5 million Canadian dollars in the account at the time of the withdrawal. It would convert the funds to U.S. dollars and transfer them to a New York bank. The relevant forward exchange rate was $0.7564. The March Canadian dollar futures contract was priced at $0.7541. Determine the outcome of a futures hedge if on February 28 the spot rate was $0.7207 and the futures rate was $0.7220. All prices are in U.S. dollars per Canadian dollar. The Canadian dollar futures contract covers CD 100,000.
15. Suppose you are a dealer in sugar. It is September 26, and you hold 112,000 pounds of sugar worth $0.0479 per pound. The price of a futures contract expiring in January is $0.0550 per pound. Each contract is for 112,000 pounds.
    a. Determine the original basis. Then calculate the profit from a hedge if it is held to expiration and the basis converges to zero. Show how the profit is explained by movements in the basis alone.
    b. Rework this problem, but assume the hedge is closed on December 10, when the spot price is $0.0574 and the January futures price is $0.0590.
16. You are the manager of a bond portfolio of $10 million face value of bonds worth $9,448,456. The portfolio has a yield of 12.25 percent and a duration of 8.33. You plan to liquidate the portfolio in six months and are concerned about an increase in interest rates that would produce a loss on the portfolio. You would like to lower its duration to 5 years. A T-bond futures contract with the appropriate expiration is priced at 72 3/32 with a face value of $100,000, an implied yield of 12 percent, and an implied duration of 8.43 years.
    a. Should you buy or sell futures? How many contracts should you use?
    b. In six months, the portfolio has fallen in value to $8,952,597. The futures price is 68 16/32. Determine the profit from the transaction.
17. (Concept Problem) As we discussed in the chapter, futures can be used to eliminate systematic risk in a stock portfolio, leaving it essentially a risk-free portfolio. A portfolio manager can achieve the same result, however, by selling the stocks and replacing them with T-bills. Consider the following stock portfolio.

Stock	Number of Shares	Price	Beta
Northrop Grumman	14,870	18.13	1.10
H. J. Heinz	8,755	36.13	1.05
Washington Post	1,245	264.00	1.05
Disney	8,750	134.50	1.25
Wang Labs	33,995	4.25	1.20
Wisconsin Energy	12,480	29.00	0.65
General Motors	14,750	48.75	0.95
Union Pacific	12,900	71.50	1.20
Royal Dutch Shell	7,500	78.75	0.75
Illinois Power	3,550	15.50	0.60

Suppose the portfolio manager wishes to convert this portfolio to a riskless portfolio for a period of one month. The price of a stock index futures with a $500 multiplier is 369.45. To sell each share would cost $20 per order plus $0.03 per share. Each company's shares would constitute a separate order. The futures contract would entail a cost of $27.50 per contract, round-trip. T-bill purchases cost $25 per trade for any number of T-bills. Determine the most cost-effective way to accomplish the manager's goal of converting the portfolio to a risk-free position for one month and then converting it back.

18. What factors must one consider when deciding on the appropriate underlying asset for a hedge?
19. State and explain two reasons why firms hedge.
20. a. Define the minimum variance hedge ratio and the measure of hedging effectiveness? What do these two values tell us?
    b. What is the price sensitivity hedge ratio? How are the price sensitivity and minimum variance hedge ratios alike? How do they differ?
21. a. What is the basis?
    b. How is the basis expected to change over the life of a futures contract?
    c. Explain why a strengthening basis benefits a short hedge and hurts a long hedge.
22. (Concept Problem) You plan to buy 1,000 shares of Swiss International Airlines stock. The current price is SF950. The current exchange rate is $0.7254/SF. You are interested in speculating on the stock but do not wish to assume any currency risk. You plan to hold the position for six months. The appropriate futures contract currently is trading at $0.7250. Construct a hedge and evaluate how your investment will do if in six months the stock is at SF926.50, the spot exchange rate is $0.7301, and the futures price is $0.7295. The Swiss franc futures contract size is SF125,000. Determine the overall profit from the transaction. Then break down the profit into the amount earned solely from the performance of the stock, the loss or gain from the currency change while holding the stock, and the loss or gain on the futures transaction.

Appendix 11

Taxation of Hedging

The tax treatment of hedging was originally established in a 1936 IRS ruling that stated that using futures contracts to reduce business risk generates ordinary income or loss. The ruling was reaffirmed in a 1955 Supreme Court case involving a firm called Corn Products Refining Company. Corn Products had purchased futures contracts to hedge the future purchase of corn it expected to need. The futures price went up and Corn Products reported the profits as capital gains, which at that time were treated more favorably for tax purposes. The IRS disagreed and the case ultimately ended up in the Supreme Court, which ruled that the purchase of the corn futures was related to the everyday operations of the firm and, thus, should be considered ordinary income for tax purposes. From that point on, the taxation of futures hedges was determined by what came to be known as the ***business motive test.*** Put simply, was the hedge designed to reduce the firm's business risk? If so, then any profits or losses would be treated as ordinary income.

This interpretation held for 33 years until a shocking ruling occurred on a case that had nothing to do with hedging. Arkansas Best, a holding company, sold shares of the National Bank of Commerce of Dallas at a substantial loss, which it reported as an ordinary loss. Ordinary losses are more attractive to the taxpaying entity because capital losses are limited to the total of capital gains. So capital losses can potentially be unusable as tax credits while ordinary losses are fully deductible against ordinary income. In 1988 the Supreme Court ruled that Best's losses were capital losses. It argued that the shares did not constitute a sufficient exception to the established definition of a capital asset. In other words, the shares were not part of Arkansas Best's inventory. Since Best was a holding company, it felt that the shares were a part of its inventory.

The IRS then began using the Arkansas Best case to argue that certain futures hedges could be treated as capital transactions, thus calling into question the millions of routine hedging transactions executed by businesses. It used the case to argue that the Federal National Mortgage Association (FNMA), a firm that buys and sells mortgages, must treat over $120 million in interest rate futures and options losses as capital. Furthermore, it ruled that while long futures and options to purchase (calls) could be viewed as substitutes for inventory positions and, thus, taxable as ordinary income, short positions and put options could never be used as substitutes for inventory because they represent contracts to sell.

Such an interpretation implied that a business holding an inventory could not reduce its risk by agreeing to sell some of the inventory in advance, at least not without potentially serious tax consequences. In other words, the IRS ruling discourages conservative business practices. The implications for the futures markets and for businesses that had routinely hedged for years were far reaching. The futures markets could be effectively shut down and millions of back taxes might be owed.

The futures exchanges and many businesses lobbied Treasury Secretary Lloyd Bentsen. Finally on October 18, 1993, the IRS reversed its ruling on the FNMA case. The IRS did argue that hedgers would need to be able to prove that futures and options transactions to protect inventory were indeed hedges. In addition, the taxation of liability hedges (such as the selling of futures in anticipation of a future issuing of liabilities), of hedges to protect the cost of raw materials purchases, and of many over-the-counter market hedges (such as the use of swaps), is still somewhat unclear.

12

SWAPS

In the last four chapters, we discussed forward and futures contracts, which are commitments for one party to buy something from another at a fixed price at a future date. In some cases a party would like to make a series of purchases, instead of a single purchase, from the other at a fixed price at various future dates. The parties could agree to a series of forward or futures contracts, each expiring at different dates. If each contract were priced according to the standard cost of carry formula, the contracts would each have a different price so that each would have a zero value at the start. A better way to construct this type of strategy, however, is to enter into a single agreement for one party to make a series of equal payments to the other party at specific dates and to receive a payment from the other party. This type of transaction, specifically characterized by a series of regularly scheduled payments, is called a swap. The parties are said to be swapping payments or assets.

Over the years, many varieties of swaps have evolved. The more common types of swaps involve one party making a series of fixed payments and receiving a series of variable payments. In addition, there are swaps in which both parties make variable payments. There are swaps in which both parties make fixed payments, but one payment is in one currency and the other is in another currency. Hence, the payments can be fixed but their values are effectively variable, given exchange rate fluctuations. The number of varieties of swaps makes it difficult to give a good all-encompassing definition, but in general, *a swap is a financial derivative in which two parties make a series of payments to each other at specific dates.*

There are four primary types of swaps, based on the nature of the underlying variable. These are currency swaps, interest rate swaps, equity swaps, and commodity swaps. In a currency swap, the parties make either fixed or variable interest payments to each other in different currencies. There may or may not be a principal payment, a point we shall cover in more detail later. In an interest rate swap, the two parties make a series of interest payments to each other, with both payments in the same currency. One payment is variable, and the other payment can be variable or fixed. The principal on which the payments are based is not exchanged. In an equity swap, at least one of the two parties makes payments determined by the price of a stock, the value of a stock portfolio, or the level of a stock index. The other payment can be determined by another stock, portfolio, or index, or by an interest rate, or it can be fixed. In a commodity swap, at least one set of payments is determined by the price of a commodity, such as oil or gold. The other payment is typically fixed, but there is no reason why it cannot be determined by some other variable. Since this book focuses on financial instruments, we shall not cover commodity swaps. For commodity swaps, see the Technical Note.

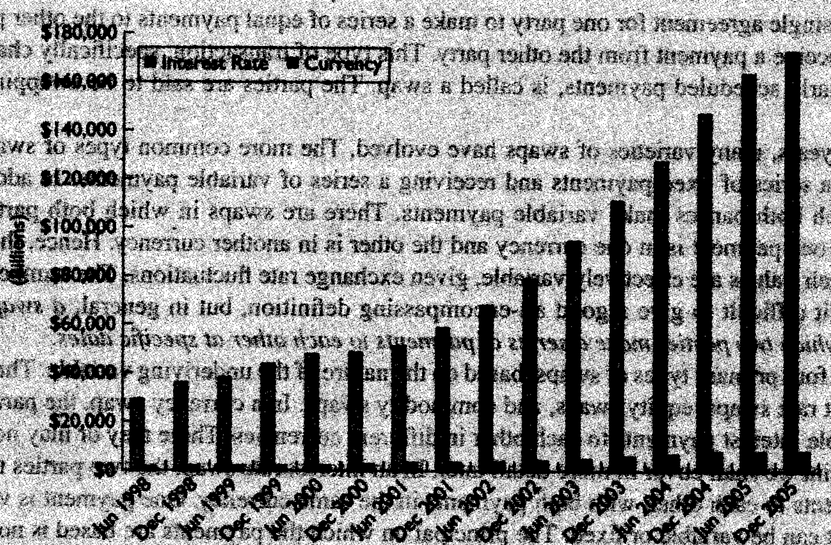
Swaps have an initiation date, a termination date, and, of course, the dates on which the payments are to be made. Like forward and futures contracts, swaps do not typically involve a cash up-front payment from one party to another. Thus, swaps have zero value at the start, which means that the present values of the two streams of payments are the same. The date on which a payment occurs is called the settlement date, and the period between settlement dates is called the settlement period. Swaps are exclusively customized, over-the-counter instruments. Thus, the two parties are usually a dealer, which is a financial institution that makes markets in swaps, and an end user, which is usually a customer of the dealer and might be a corporation, pension fund, hedge fund, or some other organization. Of course, swaps between dealers are common as well.

Swap dealers quotes prices and rates at which they will enter into either side of a swap transaction. When they do a transaction with a counterparty, they assume some risk from the counterparty. The dealers then typically hedge that risk with some other type of transaction, which could involve trading in the underlying, or using futures, forwards, or options. Interestingly, swap dealers are a major contributor to trading volume in Eurodollar futures, as they use the contract to hedge many of their interest rate swaps as well as other interest rate derivatives, which we shall cover in Chapter 13. In fact, this is the most common use of Eurodollar futures.

Like forward contracts, swaps are subject to the risk that a given party could default. Wherever possible the payments are netted, so that only a single amount is paid from one party to the other. This procedure reduces the credit risk by reducing the amount of money flowing between the parties. We shall cover credit risk in more detail in Chapter 15.

Each swap is characterized by an amount of money called the notional principal. Since currency swaps and interest rate swaps both involve making interest payments, the payments are based on the multiplication of an interest rate times a principal amount. In interest rate swaps this principal amount is never exchanged.[1] For that reason, it is not called principal, but rather notional principal.[2]

FIGURE 12.1 Notional Principal of Interest Rate and Currency Swaps



Source: Bank for International Settlements, www.bis.org

[1]In addition, notional principal is not exchanged in an equity swap. In a currency swap, notional principal may or may not be exchanged. We shall cover this point later in this chapter.

[2]In this context the word *notional* means imaginary. As noted, the principal is never paid in an interest rate or equity swap and for that reason the principal is considered to not be real. In some currency swaps, the principal is paid, but the term *notional* is still used.

Swaps have been one of the greatest success stories in the financial markets of the 1980s and 1990s. Interest rate swaps, for example, are widely used by corporations to manage interest rate risk. As we shall see, corporations often convert floating-rate loans to fixed-rate loans using interest rate swaps. Currency and equity swaps are used far less than interest rate swaps, but they are still important tools for managing currency and equity risk, respectively. Figure 12.1 shows the notional principal of interest rate and currency swaps from 1998 through the end of 2005, taken from the semiannual surveys of the Bank for International Settlements (http://www.bis.org). There has been steady growth in the use of interest rate swaps, which had notional principal at the end of 2005 of almost $173 trillion. Currency swaps have not experienced as much growth. Their notional principal at the end of 2005 was about $8.5 trillion.[3] The reason that interest rate swaps are more widely used than currency swaps is that virtually every business borrows money and is, therefore, exposed to some form of interest rate risk. Even if a business borrows at a fixed rate, changes in interest rates create opportunity costs. Many businesses are exposed to currency risk, either through their international operations, their international customers or suppliers, or from foreign competitors who offer similar products and services and sell those services in the business's home market. Nonetheless, far more firms are exposed to and understand the implications of interest rate risk than currency risk. Hence, interest rate swaps are more widely used than currency swaps in managing risk.

This chapter is divided into three main sections, each based on the three different types of swaps characterized by the underlying. For each type of swap, we shall examine the basic characteristics of the instruments, learn how to set the terms of the swap (a process called pricing), learn how to find the market value of the swap, and examine some strategies using the swap.

INTEREST RATE SWAPS

As we previously described, an interest rate swap is a series of interest payments between two parties. Each set of payments is based on either a fixed or a floating rate. If the rate is floating and the swap is in dollars, the rate usually employed is dollar LIBOR. Swaps in other currencies use comparable rates in those currencies. The two parties agree to exchange a series of interest payments in the same currency at the various settlement dates. The payments are based on a specified notional principal, but the parties do not exchange the notional principal since this would involve each party giving the other party the same amount of money.

The most common type of interest rate swap, indeed the most common type of all swaps, is a swap in which one party pays a fixed rate and the other pays a floating rate. This instrument is called a plain vanilla swap, and sometimes just a vanilla swap. Let us look at an example of a plain vanilla swap. At this point, however, we shall not concern ourselves with the reason why a company enters into a swap. Our goal right now is to understand the basic characteristics of the transaction.

Structure of a Typical Interest Rate Swap

Consider a firm called XYZ that enters into a $50 million notional principal swap with a dealer called ABSwaps. The initiation date is December 15. The swap calls for ABSwaps to make payments to XYZ based on 90-day LIBOR on the 15th of March, June, September, and December for one year. The payment is determined by three-month LIBOR at the beginning of the settlement period. Payment is then made at the end of the settlement period. Thus, the payment on March 15 is based on LIBOR on the previous December 15, and the payment on June 15 is based on LIBOR on the previous March 15, and so forth. This rate setting and

[3]Equity swaps are not shown because the market is so small relative to even currency swaps. In addition, the Bank for International Settlements combines forwards with swaps in reporting the figures for the equity markets. The notional principal of equity swaps and forwards at the end of 2004 was about one-tenth that of currency swaps.

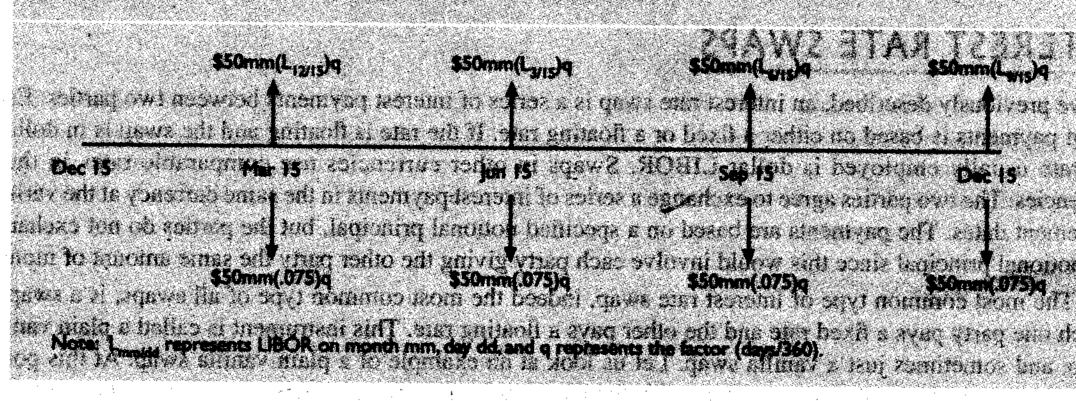
settlement procedure is known as advanced set, settled in arrears.[4] XYZ will pay ABSwaps fixed payments a rate of 7.5 percent per year. The interest payments can be based on the exact day count between payment dates, or the parties could assume 30 days in a month. Here we assume they use the exact day count. In addition, the parties could base the payment on a 360- or 365-day year. We assume that they use 360 days. In the general case, the party paying a fixed rate and receiving a floating rate will have a cash flow at each interest payment date of

$$(\text{Notional Principal})(\text{LIBOR} - \text{fixed rate})\left(\frac{\text{Days}}{360 \text{ or } 365}\right)$$

where it is understood that LIBOR is determined on the previous settlement date (advanced set, settled in arrears).

The fraction of the year calculation is known as the accrual period. There are many different ways to compute the accrual period. You can compute the actual number of days between two dates or use a method known as "30/360" day count method. The treatment of weekend and holidays also varies. The number of days in a year also can vary, including 360 days, 365 days, actual number of days, and so forth. Whether the swap is advanced set, advanced settled, or advanced set, settled in arrears—as well as the way in which the accrual period is computed—are important considerations when designing and valuing interest rate swaps. These technical aspects of swap design are a combination of elements: the convention within a particular market for the underlying instrument, the manner in which the dealer customarily quotes swaps, and the needs or preferences of the end user.

FIGURE 12.2 Cash Flows in Plain Vanilla Interest Rate Swap from Point of View of XYZ Company



Throughout this chapter we will assume advanced set, settled in arrears and that the accrual period is the actual number of days divided by 360. From the perspective of XYZ, the payments are

$$\$50{,}000{,}000(\text{LIBOR} - 0.075)\left(\frac{\text{Days}}{360}\right).$$

So if LIBOR exceeds 7.5 percent, XYZ will receive a payment in the above amount from ABSwaps. If LIBOR is less than 7.5 percent, XYZ will make a payment to ABSwaps. As we previously mentioned, to reduce the flow of money, which reduces the credit risk, the two parties agree to net the payments. Thus, one party makes a net payment to the other.

Figure 12.2 shows the cash flows on the swap from the perspective of XYZ. Note that LIBOR on the previous date determines the payment. Hence, when the swap is initiated, LIBOR on December 15

[4]An alternative rate setting and settlement procedure is known as advanced set, advanced settled, where payment is determined by LIBOR at the beginning of the settlement period and payment is also made at the beginning of the settlement period. This type of settlement helps to lower credit risk.

determines the floating payment on March 15. Then on March 15, LIBOR on that day determines the payment on June 15. In this manner, the parties always know the upcoming floating payment, but they do not know any floating payments beyond the next one.

Now let us make an assumption about the interest rates that prevail over the life of the swap in order to calculate the payments in the swap. Suppose LIBOR on December 15 is 7.68 percent. The first payment occurs on March 15. Assuming that there is no leap year, there are 90 days between December 15 and March 15. Thus, on March 15, ABSwaps will owe

$$\$50{,}000{,}000(0.0768)\left(\frac{90}{360}\right) = \$960{,}000.$$

XYZ will owe

$$\$50{,}000{,}000(0.075)\left(\frac{90}{360}\right) = \$937{,}500.$$

Table 12.1 After-the-Fact Payments in Plain Vanilla Interest Rate Swap

Notional principal:	$50,000,000
Fixed rate:	7.5%
Accrual Period:	Actual day count and 360-day year
Settlement:	Advanced set, settled in arrears

Date	LIBOR (%)	Days in Period	ABSwaps Owes	XYZ Owes	Net to XYZ
Dec 15	7.68				
Mar 15	7.50	90	$960,000	$937,500	$22,500
Jun 15	7.06	92	958,833	958,833	0
Sep 15	6.06	92	902,111	958,833	−56,222
Dec 15		91	765,917	947,917	−182,000

Note: This combination of LIBORs on the above dates represents only one of an infinite number of possible outcomes to the swap. They are used only to illustrate how the payments are determined and not the likely results.

Since the two parties agree to net the payments, only the difference of $22,500 is paid by ABSwaps to XYZ. Now let us assume that LIBOR on March 15 is 7.50 percent, on June 15 is 7.06 percent, and on September 15 is 6.06 percent. Table 12.1 illustrates the payments in this swap. Remember, however, that the floating payments on June 15, September 15, and December 15 were not known when the swap was initiated, and we merely assumed a series of interest rates over the life of the swap for illustrative purposes. The actual interest rates and payments could be quite different from these. Note also that the parties never exchange the notional principal, because this would be unnecessary.

On some occasions the parties could specify that both payments be floating. Suppose that instead of paying 7.5 percent fixed, XYZ would prefer to pay a floating rate based on the Treasury bill rate. Of course XYZ receives a floating rate based on LIBOR. Since LIBOR is the rate paid by a London bank, which is a private borrower, and the Treasury bill rate is the rate paid by the U.S. government, LIBOR will always be more than the Treasury bill rate. Obviously XYZ would prefer to receive a higher rate, but ABSwaps would not agree to such a transaction because it would always lose money. ABSwaps would be willing to do the transaction, however, by adding a spread to the Treasury bill rate or deducting a spread from LIBOR. This type of swap is called a basis swap, because the underlying is the basis risk in the relationship between LIBOR and the Treasury bill rate. In order to determine an appropriate spread, we must learn how to determine the prices and values of swaps.

Pricing and Valuation of Interest Rate Swaps

In the example of the plain vanilla swap in the previous section, we assumed a fixed rate of 7.5 percent. This is not an arbitrary rate. The dealer, ABSwaps, determined this rate by taking into account current interest rates and its ability to hedge the risk associated with this transaction. Just as we priced options, forwards, and futures by eliminating the opportunity to earn an arbitrage profit, we must do similarly with swaps. For plain vanilla swaps, we must determine the fixed rate in a process called pricing the swap. We do this in such a manner that there is no opportunity to earn an arbitrage profit. Recalling that a swap has zero value at the start, we determine the fixed rate so that the present value of the stream of fixed payments is the same as the present value of the stream of floating payments at the start of the transaction. Thus, the obligations of one party have the same value as the obligations of the other at the start of the transaction.

In order to understand interest rate swap pricing, we must first take a slight digression and look at floating rate bonds. A floating rate bond is one in which the coupons change at specific dates with the market rate of interest. Typically the coupon is set at the beginning of the interest payment period, interest then accrues at that rate, and the interest is paid at the end of the period. The coupon is then reset for the next period. The coupon is usually determined by a formula that defines it as a specific market rate, such as LIBOR, plus a spread to reflect the credit risk. Since we are not addressing credit risk at this stage of the book, we shall assume a zero credit spread.

Suppose we are given a term structure of interest rates of $L_0(t_1), L_0(t_2), \ldots, L_0(t_n)$ where the Ls represent LIBOR for maturities of t_1 days, t_2 days, and so forth, up to t_n days. Thus, if we are looking out two years at quarterly intervals, t_1 might be 90, t_2 might be 180, t_8 might be 720. Let $B_0(t_1)$ be the price of a \$1 discount (zero coupon) bond based on the rate $L_0(t_1)$; a similar pattern applies for other discount bond prices. Thus,

$$B_0(t_1)=\frac{1}{1+L_0(t_1)\left(\frac{t_1}{360}\right)},\quad B_0(t_2)=\frac{1}{1+L_0(t_2)\left(\frac{t_2}{360}\right)},\ldots,\quad B_0(t_n)=\frac{1}{1+L_0(t_n)\left(\frac{t_n}{360}\right)}.$$

In other words, the bond price for a maturity of t_1 days is the present value of \$1 in t_1 days, using the LIBOR method of discounting, which is based on a bond with add-on interest. Thus, these zero coupon bond prices can be viewed as present value factors, so we can use them to discount future payments.

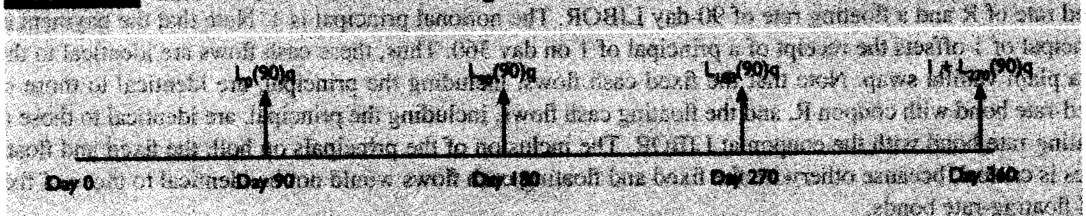
Consider a one-year floating-rate bond, with interest paid quarterly at LIBOR, assuming 90 days in each quarter. Assume a par value of 1. Thus, at time 0, 90-day LIBOR is denoted as $L_0(90)$. Ninety days later, 90-day LIBOR is denoted as $L_{90}(90)$, and $L_{180}(90)$ and $L_{270}(90)$ are the 90-day LIBORs over the remainder of the life of the loan. Of course, only $L_0(90)$ is known at the start. The party buying this floating-rate bond receives the payments shown in Figure 12.3, where $q = 90/360$.

Note the payment at the maturity date, day 360, of the principal plus the interest of $L_{270}(90)(90/360)$. Now step back to day 270 and determine the value of this upcoming payment. To determine this value, we would discount $1 + L_{270}(90)(90/360)$ using an appropriate one-period discount rate, which is $L_{270}(90)$. Let us denote the value of this floating rate bond on day 270 as $FLRB_{270}$, which is obtained as

$$FLRB_{270}=\frac{1+L_{270}(90)(90/360)}{1+L_{270}(90)(90/360)}=1$$

Hence, the value of the floating-rate bond on day 270 is its par value of 1. Now step back to day 180 and determine the value of the floating-rate bond. From day 180 and looking ahead to day 270, the holder of the bond knows that he will receive a coupon of $L_{180}(90)(90/360)$ and will hold a bond worth $FLRB_{270} = 1$, which the equation above tells us is equal to 1. He will then discount these two values at the appropriate one-period rate $L_{180}(90)$. Thus,

FIGURE 12.3 Cash Flows in One-Year Floating-Rate Bond with Quarterly Payments at LIBOR



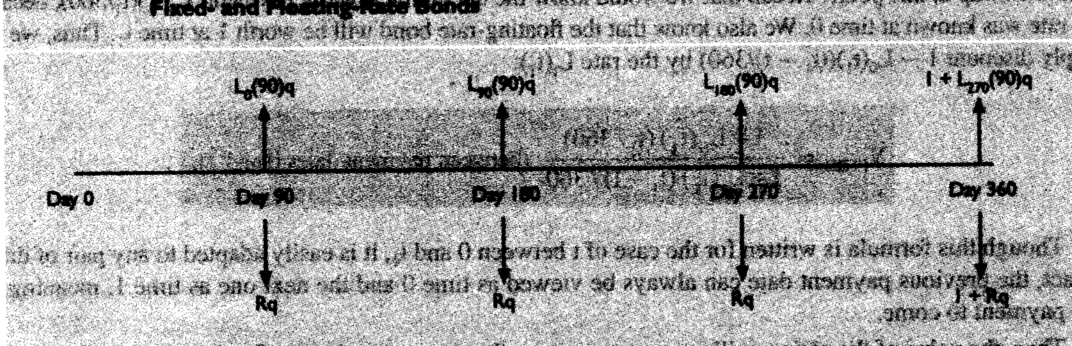
Note: $L_0(90)$, for example, is 90-day LIBOR on day 0, and q represents the factor (days/360).

$$FLRB_{180} = \frac{1 + L_{180}(90)(90/360)}{1 + L_{180}(90)(90/360)} = 1$$

Continuing this procedure back to day 0 shows that the value of the floating-rate bond at any payment date, as well as on the initial date, is 1, its par value. This will always be the case if the coupon does not contain a spread over LIBOR. A floating-rate bond is designed such that its price will stay at or near par value. Between the interest payment dates, its price can stray from par value but it would take a very large interest rate change for it to deviate much from par value. ***The result that the value of a floating-rate bond is par at its payment date is an extremely important one, and one upon which we shall rely heavily when pricing and valuing interest rate swaps.***[5]

Now let us consider what we mean by the value of a swap. Look back at Figure 12.2. A plain vanilla swap is a stream of fixed interest payments and a stream of floating interest payments. The fixed interest payments are similar to those of a fixed-rate bond, except that a fixed-rate bond would pay back its principal at maturity. Likewise, the floating interest payments are similar to those of a floating-rate bond, except that the floating-rate bond would pay back its principal at maturity. An interest rate swap, of course, does not involve principal payments. Suppose, however, that we add and subtract the notional principal at the termination date of the swap. The cash flows would still be the same as those on a swap, but now we could view the fixed payments as though they were the cash flows on a fixed-rate bond, and the floating payments as though they were the cash flows on a floating-rate bond.

FIGURE 12.4 Cash Flows in One-Year Quarterly Plain Vanilla Interest Rate Swap Decomposed into Fixed- and Floating-Rate Bonds



Note: $L_0(90)$, for example, is 90-day LIBOR on day 0, R is the fixed rate, and q represents the factor (days/360).

[5]Clearly, credit risk changes may cause floating rate bonds not to trade at par. Credit risk changes and other technical nuances are beyond the scope of this introductory book.

Consider Figure 12.4, which depicts the cash flows on the one-year quarterly swap, assuming a general fixed rate of R and a floating rate of 90-day LIBOR. The notional principal is 1. Note that the payment of a principal of 1 offsets the receipt of a principal of 1 on day 360. Thus, these cash flows are identical to those on a plain vanilla swap. Note that the fixed cash flows, including the principal, are identical to those of a fixed-rate bond with coupon R, and the floating cash flows, including the principal, are identical to those of a floating-rate bond with the coupon at LIBOR. The inclusion of the principals on both the fixed and floating sides is critical, because otherwise the fixed and floating cash flows would not be identical to those of fixed- and floating-rate bonds.

It should now be apparent that a pay-fixed, receive-floating swap is equivalent to issuing a fixed-rate bond and using the proceeds to purchase a floating-rate bond. Thus, the value of a swap to pay a fixed rate and receive a floating rate is equal to the value of the floating-rate bond minus the value of the fixed-rate bond.

To make these results a little more general, let us consider a swap with n payments, made on days $t_1, t_2, \ldots, t_n$. The value of the corresponding fixed-rate bond, V_{FXRB}, with coupon R is easy to determine:

$$V_{FXRB} = \sum_{i=1}^{n} R\left(\frac{t_i - t_{i-1}}{360}\right) B_0(t_i) + B_0(t_n)$$

where $B_0(t_i)$ is the discount factor, as discussed above, for the period 0 to day t_i. In other words, it is the value of an m-day zero coupon bond with maturity on day t_i. Each coupon $R((t_i - t_{i-1})/360)$ is multiplied by the discount factor. In addition, the final principal of 1 (omitted in the equation above) is multiplied by the discount factor for day t_n. The above expression is simply the present value of the interest and principal payments on a fixed-rate bond.

The present value of the floating-rate bond at time 0 is extremely simple. Now, we must recall what we learned about a floating-rate bond. That value at any coupon date as well as at the start is the par value, here 1. Thus, the value of the floating-rate bond is

$$V_{FLRB} = 1 \text{ (at time 0 or a payment date)}$$

Suppose we are at a date such as time t, which we shall position between time 0 and time t_1. How would we value the swap at this point? Recall that we would know the upcoming floating payment, $L_0(t_1)(t_1/360)$, because this rate was known at time 0. We also know that the floating-rate bond will be worth 1 at time t_1. Thus, we can simply discount $1 - L_0(t_1)((t_1 - t)/360)$ by the rate $L_t(t_1)$:

$$V_{FLRB} = \frac{1 + L_0(t_1)(t_1/360)}{1 + L_t(t_1)((t_1 - t)/360)}$$

Though this formula is written for the case of t between 0 and t_1, it is easily adapted to any pair of dates. In fact, the previous payment date can always be viewed as time 0 and the next one as time 1, meaning the first payment to come.

Thus, the value of the plain vanilla swap to receive a floating rate and pay a fixed rate is

$$VS = V_{FLRB} - V_{FXRB}$$

This result is based on a notional principal of 1. For any other notional principal, simply multiply VS by the notional principal. Also note that from the counterparty's perspective, the value is found by subtracting V_{FLRB} from V_{FXRB}. This, of course, would be the value of the plain vanilla swap for paying a floating rate and receiving a fixed rate.

This formulation takes the fixed rate, R, as already known. At the beginning of the life of the swap, the fixed rate is set such that the value of the swap is zero. In this manner, each party's obligation to the other is the same. Hence, the swap has zero value to both parties, and neither pays the other anything at the start. To establish the value of R at the start, we must solve for R such that the present value of the stream of fixed payments plus the hypothetical notional principal equals 1, which is the present value of the stream of floating payments plus the hypothetical notional principal. Thus, R is the coupon rate on a par value bond. We find R by setting VS to zero and solving for R to obtain (where $q = ((t_i - t_{i-1})/360)$, which we assume is constant for all i)):

$$R = \left(\frac{1}{q}\right)\left(\frac{1 - B_0(t_n)}{\sum_{i=1}^{n} B_0(t_i)}\right)$$

This is a simple calculation to perform. An example is illustrated in Table 12.2.

Swap dealers perform this calculation to determine the fixed rate, but they typically quote the rate to their customers in a different manner. To make money, dealers must pay a lower fixed rate when they enter into a pay-fixed, receive-floating swap, and receive a higher fixed rate when they enter into a pay-floating, receive-fixed swap. For example, the dealer might quote a rate of 9.79 percent for a swap in which the dealer receives a fixed rate and 9.71 percent for a swap in which the dealer pays a fixed rate.

Many dealers quote their rates electronically and make them available through various data service providers. However, sometimes the dealer does not quote the receive fixed or receive floating swap rates in this form. For example, to quote either rate on a two-year swap, the dealer determines the rate on a two-year

Table 12.2 Pricing a Plain Vanilla Interest Rate Swap

Scenario: Quantum Electronics enters into a two-year \$20 million notional principal interest rate swap in which it promises to pay a fixed rate and receive payments at LIBOR. The payments are made every six months based on the assumption of 30 days per month and 360 days in a year. The term structure of LIBOR interest rates and the zero coupon bond prices based on those rates are given as follows:

Term	Rate	Discount Bond Price
180 days	$L_0(180) = 9.00\%$	$B_0(180) = 1/(1 + 0.09(180/360)) = 0.9569$
360 days	$L_0(360) = 9.75\%$	$B_0(360) = 1/(1 + 0.0975(360/360)) = 0.9112$
540 days	$L_0(540) = 10.20\%$	$B_0(540) = 1/(1 + 0.1020(540/360)) = 0.8673$
720 days	$L_0(720) = 10.50\%$	$B_0(720) = 1/(1 + 0.1050(720/360)) = 0.8264$

With $q = 180/360$, then $1/q = 360/180$ and the fixed rate would, therefore, be

$$R = \left(\frac{360}{180}\right)\left(\frac{1 - 0.8264}{0.9569 + 0.9112 + 0.8673 + 0.8264}\right) = 0.0975.$$

Thus, the rate would be 9.75 percent. The swap fixed payments would be

$$\$20,000,000(0.0975)(180/360) = \$975,000.$$

government security, such as a U.S. Treasury note. Suppose this rate is 9.60 percent. Then 9.79 percent would be quoted as the 2-year Treasury rate plus 19, and 9.71 percent would be quoted as the 2-year Treasury rate plus 11. This procedure enables the dealer to make a quote that will hold up for a period of time. If interest rates make a quick move, the Treasury rate will move and the quoted swap rate will still be aligned at a fixed spread over the Treasury rate.

The spread of the swap rate over the corresponding Treasury rate is referred to as the swap spread and reflects the general level of credit risk in the global economy. That is, LIBOR is a borrowing rate that reflects the credit risk of London-based banks. The Treasury note rate reflects the default-free borrowing rate of the U.S. government as well as several other unique features like being exempt from state and local taxes. When the economy weakens, credit risk becomes greater and the spread between LIBOR and the Treasury rate widens, leading to a larger swap spread.

As we discussed, the swap would have a value of zero at the start. As soon as interest rates change or time elapses, however, the swap will move to a nonzero value. In other words, its value to one party will be positive, and its value to the other will be negative. Table 12.3 illustrates how the Quantum Electronics swap is valued three months into its life, or half-way between the initiation date and the first payment date.

Valuation of a swap is extremely important. If a firm enters into a swap, it knows that, ignoring the dealer's spread, the swap has a zero value at the start. It is neither an asset nor a liability. Once time elapses or interest rates change, however, the value of the swap will move to a positive value for one party and a negative value for the other. If the swap has a positive value, it is an asset. If it has a negative value, it is a liability. Proper accounting practice requires that swaps be valued, their gains and losses shown on the balance sheet.[6] Moreover, any financial officer responsible for a firm's swaps would want to know the values of its swap to determine how well the transaction is performing. Another reason why valuation is important is that it is a measure of the credit risk in a transaction. Suppose your firm holds a swap worth $100,000.

Table 12.3 Valuing a Plain Vanilla Interest Rate Swap During its Life

When the Quantum Electronics swap was first established, the first floating payment was set at the 180-day rate of 9 percent. For a $1 notional principal, the payment would be 0.09(180/360) = 0.045. The fixed payment is at 9.75 percent, so it would be 0.0975(180/360) = 0.04875. To value the swap 90 days into its life, we need the new term structure of interest rates as follows:

Term	Rate	Discount Bond Price
90 days	$L_{90}(90) = 9.125\%$	$B_{90}(90) = 1/(1 + 0.09125(90/360)) = 0.9777$
270 days	$L_{90}(270) = 10.000\%$	$B_{90}(270) = 1/(1 + 0.10(270/360)) = 0.9302$
450 days	$L_{90}(450) = 10.375\%$	$B_{90}(450) = 1/(1 + 0.10375(450/360)) = 0.8852$
630 days	$L_{90}(630) = 10.625\%$	$B_{90}(630) = 1/(1 + 0.10625(630/360)) = 0.8432$

The value of the fixed payments, including the hypothetical notional principal, is

$$V_{FXRB} = 0.04875(0.9777 + 0.9302 + 0.8852 + 0.8432) + 1.0(0.8432) = 1.02046963.$$

The value of the floating payments, including the hypothetical notional principal, is based on discounting the next floating payment of 0.045 and the market value of the floating-rate bond on the next payment date, which is 1:

$$V_{FLRB} = (0.045 + 1.0)(0.9777) = 1.0216965.$$

Thus, the value of the swap per $1 notional principal is

$$VS = 1.0216965 - 1.02046963 = 0.00122687.$$

Taking into account the $20 million notional principal, the value of the swap is

$$\$20{,}000{,}000(0.00122687) = \$24{,}537.$$

To the counterparty, the value of the swap is −$24,537.

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[6]We shall cover derivatives accounting in Chapter 16.

Because the obligations of the counterparty exceed the obligations of your firm by $100,000, you are susceptible to losing $100,000 if the counterparty defaults.[7] Yet another reason why we would want to know the value of the swap is that we might wish to terminate the swap position. We could do this by selling it back to the counterparty if it has a positive value or buying it back from the counterparty if it has a negative value. We shall discuss this procedure later in this chapter.

Thus, having a position in a swap and determining its value is, in principal no different from owing a stock, looking into the market, and seeing what its price is. More calculations are required with swaps, because unlike stock, a swap is a customized instrument that does not trade in an open market where its value can be read off of a computer screen. Nonetheless, the information necessary for valuing the swap can be observed in the market, and the calculations can be easily made. Alternatively, an end user can simply ask the dealer counterparty for a valuation at any time during the life of the swap.

We have been discussing the pricing and valuation of plain vanilla interest rate swaps. Recall that we also briefly discussed the basis swap, which is a swap in which both sides make floating payments. One common type of basis swap is where one side pays the Treasury rate and the other pays LIBOR. As we noted, the side paying LIBOR would always be paying the higher rate so it would need to be compensated with a fixed spread. To price this swap, let us make the present value of the payments at the Treasury bill rate equal the present value of the payments at LIBOR by incorporating a spread. Consider the following:

A swap to

pay the T-bill rate

receive a fixed rate derived from the T-bill term structure

Plus,

A swap to

pay a fixed rate derived from the LIBOR term structure

receive LIBOR

Equals

a swap to pay the T-bill rate and receive LIBOR, and pay the difference between the LIBOR fixed rate and the T-bill fixed rate.

Thus, a basis swap to pay the T-bill rate and receive LIBOR will also involve paying the difference between the LIBOR fixed rate and the T-bill fixed rate. This should make sense from a logical point of view: LIBOR is greater than the T-bill rate so the party receiving LIBOR would have to give up something. Table 12.4 illustrates the pricing of this swap. In it we price a swap using both the T-bill term structure and the LIBOR term structure. The spread between these fixed rates is the spread on a basis swap.

Now suppose we move three months into the life of the swap and wish to obtain its value. Table 12.5 shows how to value the basis swap during its life.

Interest Rate Swap Strategies

Now let us return to the example we used at the beginning of the chapter. Recall that a firm called XYZ entered into a swap to pay a fixed rate and receive a floating rate. Now, we want to understand why XYZ would do this transaction. Suppose the current date is December 15. XYZ has a one-year floating-rate loan at LIBOR plus 100 basis points. The payments are on the 15th of March, June, September, and December, and the interest is calculated based on the actual number of days in the period divided by 360. If XYZ would prefer

[7]We shall cover credit risk in more detail in Chapter 15.

Table 12.4 Pricing a Basis Swap

Consider a one-year swap with semiannual payments to pay the T-bill rate and receive LIBOR minus a spread with payments based on days/360, assuming 30 days in a month. The notional principal is $50 million. The term structures are as follows:

Term	LIBOR	Discount Bond Price
180 days	7.01%	$B_0(180) = 1/(1 + 0.0701(180/360)) = 0.9661$
360 days	7.21%	$B_0(360) = 1/(1 + 0.0721(360/360)) = 0.9327$

Term	T-Bill Rate	Discount Bond Price
180 days	5.05%	$B_0(180) = 1 - 0.0505(180/360) = 0.9748$
360 days	5.95%	$B_0(360) = 1 - 0.0595(360/360) = 0.9405$

Note that the T-bill discount factor is determined using the discount method, while the LIBOR discount factor is determined using the add-on method. This is the convention in the two markets, as we have seen in previous chapters. Solving for the LIBOR fixed rate, we obtain

$$R = \left(\frac{360}{180}\right)\left(\frac{1-0.9327}{0.9661+0.9327}\right) = 0.0709.$$

Solving for the T-bill fixed rate, we obtain

$$R = \left(\frac{360}{180}\right)\left(\frac{1-0.9405}{0.9748+0.9405}\right) = 0.0621.$$

The spread is, thus, 0.0709 – 0.0621 = 0.0088. Thus, in this swap, the party paying the T-bill rate would pay 88 basis points more, or the party paying LIBOR would pay 88 basis points less. We shall assume that the party paying LIBOR pays LIBOR minus 88 basis points.

Table 12.5 Valuing a Basis Swap During its Life

Consider the swap described in Table 12.4. Now it is 90 days into the life of the swap. The new term structures are as follows:

Term	LIBOR	Discount Bond Price
90 days	7.20%	$B_{90}(90) = 1/(1 + 0.072(90/360)) = 0.9823$
270 days	7.35%	$B_{90}(270) = 1/(1 + 0.0735(270/360)) = 0.9478$

Term	T-bill Rate	Discount Bond Price
90 days	5.30%	$B_{90}(90) = 1 - 0.053(90/360) = 0.9868$
270 days	6.20%	$B_{90}(270) = 1 - 0.062(270/360) = 0.9535$

The present value of the floating T-bill payments can be found by discounting the upcoming payment plus the par value of the payments on the next payment date, which as we saw is the market value of the floating-rate bond on the next payment date. The next payment will be at the rate of 5.05% because this was the 180-day rate when the swap was initiated:

$$(1 + 0.0505(180/360))(0.9868) = 1.0117167.$$

The upcoming LIBOR payment will be at 7.01% minus the spread of 0.0088 = 0.0613. Then the present value of the LIBOR payments will be

$$(1 + 0.0613(180/360))(0.9823) = 1.0124075.$$

Then the value of the swap to pay the T-bill rate and receive LIBOR for a $1 notional principal will be

$$1.0124075 - 1.0117167 = 0.0006908.$$

Based on the $50 million notional principal, the value of the swap will be

$$\$50{,}000{,}000(0.0006908) = \$34{,}540.$$

a fixed-rate loan, it can easily convert the floating-rate loan into a fixed-rate loan by engaging in the swap we described. Recall that in that swap, XYZ pays a fixed rate of 7.5 percent and receives LIBOR. Figure 12.5 shows that the swap has the effect of leaving XYZ paying a fixed rate of 8.5 percent, reflecting the 7.5 percent fixed rate on the swap, plus the 100 basis point spread it pays over LIBOR on the floating-rate loan.

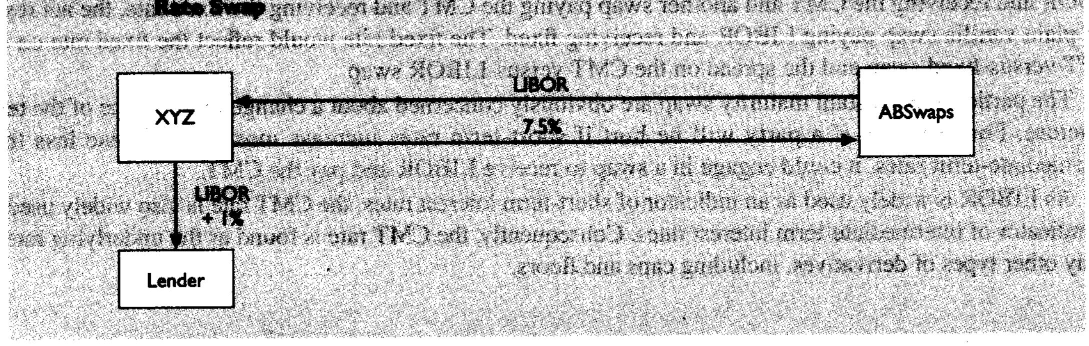
Of course, you may be wondering why XYZ did not just take a fixed-rate loan in the first place. One reason is that because they tend to borrow at floating rates, banks prefer to make floating rate loans. They will tend to charge a slight premium for fixed-rate loans. In addition, if XYZ takes a floating-rate loan and swaps it into a fixed-rate loan, it will assume some credit risk from the possibility that the ABSwaps will default. If XYZ had taken out a fixed-rate loan it would assume no credit risk, because it would not have a claim on any payments. The assumption of credit risk can result in some savings. Of course ABSwaps could default, and XYZ would still have to make its floating interest payments to its lender. The savings come at the expense of assuming the risk of ABSwaps defaulting.

This example is the most common application of interest rate swaps, particularly by corporations. Indeed it is probably one of the most common of all financial transactions. It should be easy to see that a swap can alternatively be used to convert a fixed-rate loan to a floating-rate loan. Corporations often do these types of transactions to alter their mix of fixed- to floating-rate borrowing to a more desirable level. Some, if not most, of these transactions are done in anticipation of interest rate changes. But regardless of the reason, the plain vanilla interest rate swap, combined with a position in a fixed- or floating-rate bond or loan is a widely used financial strategy.

Basis swaps are frequently used in speculative situations. Taking the example above, the spread between the LIBOR and T-bill swap rates is 88 basis points. A party might believe that the spread would widen or narrow due to changes in the perception of credit risk in the global economy. A basis swap could be used to speculate on such an occurrence. Likewise, a basis swap could be used to hedge changes in the general level of credit risk in the global economy. Such a strategy might be useful to a party holding a portfolio of credit risky bonds, who is concerned about the effects of a deterioration of credit quality in the global economy. As we shall see in Chapter 15, however, there are credit derivatives that work much better in managing credit risk.

Index Amortizing Swaps An index amortizing swap is one in which the notional principal is reduced as one moves through time. A typical application of this type of swap would be where the party holds another position in which the notional principal is designed to decline through time. The most common example of this is the mortgage loan. When a homeowner takes out a mortgage, the loan balance reduces through time according to a fixed schedule. Homeowners, however, nearly always have the option to prepay and refinance their mortgages. When interest rates fall, more homeowners exercise their options. The holders of mortgages thus suffer losses by having their income streams reduced to zero and having to reinvest the prepaid principal

FIGURE 12.5 **Conversion of Floating-Rate Loan into Fixed-Rate Loan Using Plain Vanilla Interest Rate Swap**



at lower rates. In an index amortizing swap, the notional principal declines according to a schedule that specifies an acceleration of the amortization rate if interest rates fall. This results in the swap notional principal behaving much like the mortgage notional principal. There are numerous variations of this type of swap.

Securities based on mortgages have become popular instruments and are often classified as derivatives. We discuss them later in Chapter 14.

Diff Swaps A diff swap is an interest rate swap based on the interest rates in two countries but where the payments are made in a single currency. For example, a U.S. firm might be concerned that German interest rates will increase relative to U.S. rates. It could hedge this position by purchasing a euro-denominated floating-rate note and selling a dollar-denominated floating-rate note. This would, however, be assuming unwanted currency risk. Alternatively, it could enter into a diff swap in which it receives the German interest rate and pays the U.S. interest rate, with all payments made in dollars. Thus, if German interest rates rise relative to U.S. interest rates, the swap will result in a net payment in dollars to the firm. Obviously the dealer in such a swap would incur the currency risk and would probably pass on to the party the cost of hedging that risk, but presumably the dealer could do it much cheaper.

It should be apparent that this swap is simply a currency-hedged basis swap. If the interest rates of one country are consistently higher than those in the other country, there would be a spread similar to that in a basis swap negotiated up front.

Constant Maturity Swaps A constant maturity swap is similar to a plain vanilla swap or a basis swap. One party pays a floating rate such as LIBOR and the other party pays another floating rate represented by the yield on a security with a maturity longer than the reset period. In other words, if the swap settles every six months, one party pays a rate on a security with longer than six months' maturity. Typically this is the yield on a security with a maturity in the range of two to five years. In dollar-denominated swaps, that rate is often referred to as the Constant Maturity Treasury (CMT) rate. The CMT rate is the yield on a U.S. Treasury note with a maturity closest to the maturity of interest. Naturally there is not always a Treasury note with precisely the desired maturity. In that case, however, the parties agree that the CMT rate will be interpolated from the rates on securities with maturities slightly above and below the desired maturity. Also, the Federal Reserve publishes its own estimate of the CMT rate, which is often used.

As an example, consider a three-year swap with semiannual payments in which one party pays the six-month LIBOR and the other pays the CMT on a five-year U.S. Treasury note. This is just a variation of a basis swap because both rates are floating. Depending on the shape of the term structure, a spread might also be paid by the party paying the lower rate. Constant maturity swaps can also have one party paying a fixed rate, with the other paying the CMT rate. It should be apparent that this is just a variation of a plain vanilla swap where the floating rate has a longer maturity than the standard LIBOR-based plain vanilla swap.

This observation gives rise to an important arbitrage relationship. If a party enters into a swap paying LIBOR and receiving the CMT and another swap paying the CMT and receiving the fixed rate, the net result is a plain vanilla swap paying LIBOR and receiving fixed. The fixed rate would reflect the fixed rate on the CMT-versus-fixed swap and the spread on the CMT-versus-LIBOR swap.

The parties to a constant maturity swap are obviously concerned about a change in the shape of the term structure. For example, if a party will be hurt if short-term rates increase more or decrease less than intermediate-term rates, it could engage in a swap to receive LIBOR and pay the CMT.

As LIBOR is widely used as an indicator of short-term interest rates, the CMT rate is also widely used as an indicator of intermediate-term interest rates. Consequently, the CMT rate is found as the underlying rate in many other types of derivatives, including caps and floors.

These examples are designed to give you a basic familiarity with the most popular variations of the plain vanilla swap. The number of variations actually seen in practice is much greater than we can show here. Moreover, many of these variations of interest rate swaps can also be used with commodity, currency, and equity swaps. If you have a good understanding of what we have covered so far, you should be ready to deal with the other types of swaps you may some day encounter.

Though interest rate swaps are more widely used than currency swaps, currency swaps are actually much more general instruments. This means that, as we shall see, an interest rate swap is just a special case of a currency swap.

CURRENCY SWAPS

As we previously described, a currency swap is a series of payments between two parties in which the two sets of payments are in different currencies.[8] The payments are effectively equivalent to interest payments because they are calculated as though interest were being paid on a specific notional principal. In a currency swap, however, there are two notional principals, one in each of the two currencies. In addition, in a currency swap, the notional principal can be exchanged at the beginning and at the end of the life of the swap, depending on the parties' desires. Because currency swap payments are in different currencies, they are typically not netted. Thus, the first party pays the second the amount owed, and the second party pays the first the amount owed.[9]

Structure of a Typical Currency Swap

Let us take a look at a currency swap between a hypothetical U.S. firm, Reston Technology, and a hypothetical dealer, Global Swaps, Inc. (GSI). For now, let us not concern ourselves with why Reston wishes to enter into this swap. We shall address the motivation for currency swaps in a later section. So let us assume that Reston enters into a currency swap with GSI in which it will make a series of semiannual interest payments in euros to GSI at a rate of 4.35 percent per year, based on a notional principal of €10 million. GSI will pay Reston semiannual interest in dollars at a rate of 6.1 percent for two years, based on a notional principal of $9.804 million. The two parties will exchange the notional principal at the beginning and at the end of the transaction. Thus, the following transactions take place:

At the initiation date of the swap
- Reston pays GSI $9.804 million
- GSI pays Reston €10 million

Semiannually for two years
- Reston pays GSI 0.0435(180/360)€10,000,000 = €217,500
- GSI pays Reston 0.061(180/360)$9,804,000 = $299,022

At the termination date of the swap
- Reston pays GSI €10 million
- GSI pays Reston $9.804 million

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[8]It is important to clear up some potential confusion over another similarly named transaction. In the foreign currency markets, a long position in one forward contract and a short position in a forward contract on the same currency but with a different expiration date is called an FX swap. This type of transaction is, therefore, similar to what we referred to as a spread when using futures. FX swaps and currency swaps are completely different transactions, but have similar names. We shall not cover FX swaps, but given that we covered futures spreads, the similarity should be obvious.

[9]In 2002 the CLS Bank International began operating as a currency clearinghouse bank, with the objective of allowing parties to net payments on all types of currency transactions. If this bank is successful in attracting users, netting will become commonplace on currency swaps, thereby reducing credit risk.

FIGURE 12.6 Cash Flows in a Currency Swap from the Point of View of Reston Technology

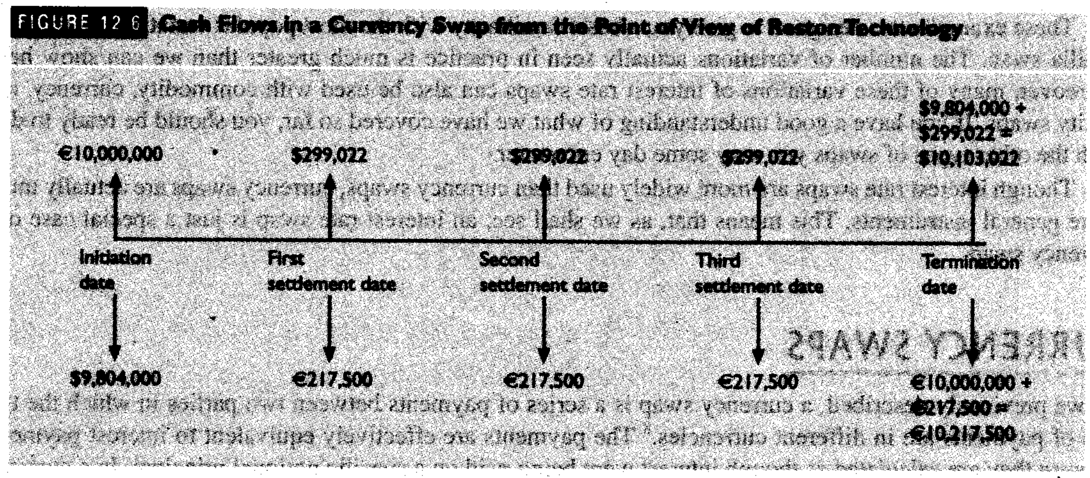


Figure 12.6 illustrates these cash flows from the point of view of Reston. Note that the series of cash flows looks like Reston has issued a euro-denominated bond for €10 million, taken the funds, and purchased a dollar-denominated bond for $9.804 million. During the two years, Reston makes payments in euros and receives payments in dollars. At the end of the two years, Reston pays back the principal of €10 million and receives the principal of $9.804 million. Of course, Reston does not actually issue a dollar-denominated bond or purchase a euro-denominated bond. It enters into a swap, but the payments are the same as if it had done the transactions in bonds. This is an important point to see in understanding how currency swaps are priced.

Note that the initial exchange of notional principals is €10 million for $9.804 million. Because we said that swaps normally have zero value at the start, the exchange rate at the time the swap is initiated is $0.9804 per euro. Of course at the end of the life of the transaction, the same €10 million is exchanged for $9.804 million, but at that time, the exchange rate will almost surely be different from $0.9804. This exchange rate risk gives rise to gains and losses for the two parties, which is an important factor in determining the value of the swap.

In this swap, both sets of payments are at a fixed rate, but both could be at a floating rate or either could be fixed and the other be floating. If a floating rate were used, the dollar floating rate would probably be LIBOR. The euro floating rate would probably be Euribor, which is the rate at which banks lend euros to each other in Frankfurt, Germany, the financial center of the European Union.

There are some important relationships between interest rate and currency swaps that we must understand. Recall that there are four types of currency swaps: (1) paying both currencies at fixed rates, (2) paying both at floating rates, (3) paying the first currency at a floating rate and the second at a fixed rate, and (4) paying the second currency at a floating rate and the first at a fixed rate. Using our example of dollars and euros as the two currencies, Figure 12.7 illustrates how currency swaps can be combined to equal interest rate swaps. We can combine two currency swaps to produce a plain vanilla interest rate swap. Likewise, we could combine a currency swap and an interest rate swap to produce another currency swap. For example, consider the first combination in Figure 12.7. Suppose we combine the swap to pay € fixed and receive $ fixed with the interest rate swap to pay $ fixed and receive $ floating. This will produce a currency swap to pay € fixed and receive $ floating. The relationships illustrated in Figure 12.7 are like an algebraic equation. The terms can be rearranged by changing "pay" to "receive" and vice versa when moving a transaction to the other side of the equality.

Perhaps the most important relationship between interest rate and currency swaps, however, is the simple fact that an interest rate swap is just a currency swap involving one side paying floating and the other paying fixed, but where both currencies are the same. Thus, the currency swap is far more general than the interest rate swap. The currency swap contains the interest rate swap as a special case.

Pricing and Valuation of Currency Swaps

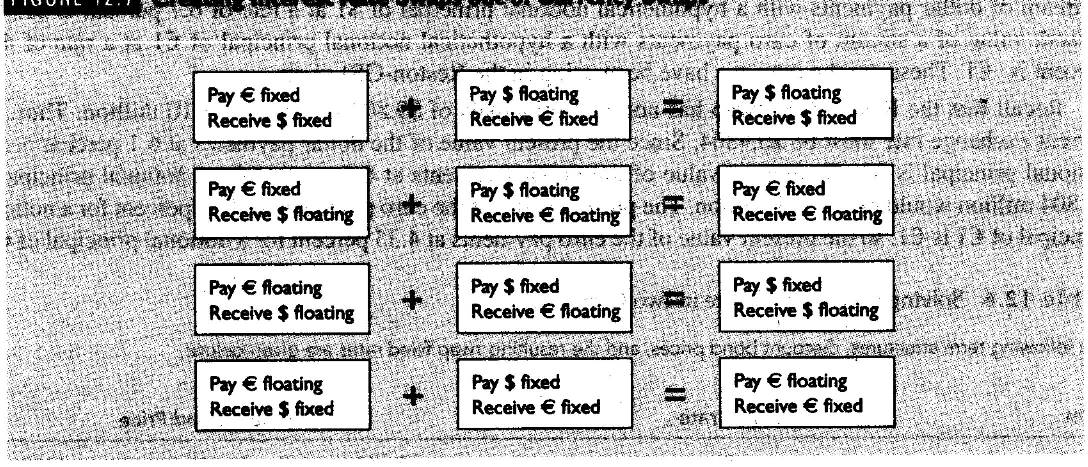
A currency swap is a transaction with two streams of cash flows, one in one currency and one in another currency. Each cash flow stream is based on a different amount of notional principal and each can be at a fixed or a floating rate. To determine the value of a currency swap, we must find the present values of the two streams of cash flows, with both expressed in a common currency. We subtract the value of the outflow stream from the value of the inflow stream.

Let us start by considering a currency swap with payments in dollars and euros. Let the dollar notional principal be $NP^{\$} = 1$ and the euro notional principal be expressed as $NP^{€}$, but the amount is initially unspecified. Since the value of the swap should be zero at the start, $NP^{€}$ will be related to $NP^{\$}$ by the current exchange rate. Let S_0 be the exchange rate, expressed as dollars per euro. Then

$$NP^{€} = 1/S_0$$

for every dollar of notional principal.

FIGURE 12.7 Creating Interest Rate Swaps out of Currency Swaps



In plain vanilla interest rate swaps, we do not exchange the notional principal.[10] In a currency swap, the exchange of notional principals is perfectly normal. The two parties exchange $NP^{\$}$ and $NP^{€}$ at the start. This exchange has no value as the two amounts are equivalent. At the end of the life of the swap, the parties reverse the original exchange. At that point, however, the exchange rate is not likely to be S_0 so the exchange is not for equivalent value. In some currency swaps, the notional principals are not exchanged. In the examples here, we shall assume that the notional principals are exchanged. Later, we shall discuss a situation in which it would be desirable to not exchange the notional principals.

For our swap with fixed payments in dollars and euros, we first need to determine the fixed rate in dollars that will make the present value of the payments equal the notional principal of \$1. Fortunately we already know that rate, because we determined it when pricing the plain vanilla interest rate swap. It is simply the fixed rate on a plain vanilla swap using the term structure in dollars. Let us denote that rate as $R^{\$}$.

Now we need to determine the fixed rate in euros that will make the present value of the fixed payments in euros equal the notional principal of $NP^{€}$. Of course, we have not yet determined $NP^{€}$. But first let us note that if we constructed a plain vanilla swap in euros with a notional principal of €1, the fixed rate would be found the same way we found the fixed rate in dollars. We would simply use the euro term structure. Let this

[10]Remember, however, that we assumed a *hypothetical* exchange of equivalent notional principals so that the two streams of cash flows would be equivalent to those of fixed- and floating-rate bonds, thereby permitting us to value the swap as though it were a long position in one bond and short position in the other.

rate be denoted as $R^€$. The present value of the fixed payments in euros at this rate would be €1. The value of the euro payments for the actual swap would be found by discounting them at a rate of $R^€$ and then multiplying by the notional principal of $NP^€$.

If either set of payments is at a floating rate, we can find their present value using the same method we used for plain vanilla interest rate swaps. We discount the next floating payment, which will be known because it has already been established, and also the value of the swap at the next payment date, which is known to be the notional principal of 1 (either dollars or euros). After obtaining the values of both sets of payments in their respective currencies and multiplying each by the notional principal, we then convert them to a common currency.

Let us now work a problem. Recall the Reston-GSI swap that we previously discussed. Table 12.6 presents the term structures in dollars and euros and solves for the fixed rate on plain vanilla swaps in both currencies. Recall that we are interested in a currency swap involving semiannual payments for two years.

We see in Table 12.6, that a plain vanilla interest rate swap in dollars would be at a rate of 6.1 percent, while a plain vanilla interest rate swap in euros would be at a rate of 4.35 percent. Thus, the present value of a stream of dollar payments with a hypothetical notional principal of $1 at a rate of 6.1 percent is $1. The present value of a stream of euro payments with a hypothetical notional principal of €1 at a rate of 4.35 percent is €1. These are the rates we have been using in the Reston-GSI swap.

Recall that the Reston-GSI swap has notional principals of $9.804 million and €10 million. Thus, the current exchange rate must be $0.9804. Since the present value of the dollar payments at 6.1 percent per $1 notional principal is $1, the present value of the dollar payments at 6.1 percent for a notional principal of $9.804 million would be $9.804 million. The present value of the euro payments at 4.35 percent for a notional principal of €1 is €1, so the present value of the euro payments at 4.35 percent for a notional principal of €10

Table 12.6 Solving for the Fixed Rate in Two Currencies

The following term structures, discount bond prices, and the resulting swap fixed rates are given below:

Term	Dollar rate	Discount Bond Price
180 days	5.5%	$B_0^\$(180) = 1/(1 + 0.055(180/360)) = 0.9732$
360 days	5.5%	$B_0^\$(360) = 1/(1 + 0.055(360/360)) = 0.9479$
540 days	6.2%	$B_0^\$(540) = 1/(1 + 0.062(540/360)) = 0.9149$
720 days	6.4%	$B_0^\$(720) = 1/(1 + 0.064(720/360)) = 0.8865$

The fixed rate on a dollar plain vanilla interest rate swap would be

$$R^\$ = \left(\frac{360}{180}\right)\left(\frac{1-0.8865}{0.9732+0.9479+0.9149+0.8865}\right) = 0.061.$$

Term	Euro rate	Discount Bond Price
180 days	3.8%	$B_0^€(180) = 1/(1 + 0.038(180/360)) = 0.9814$
360 days	4.2%	$B_0^€(360) = 1/(1 + 0.042(360/360)) = 0.9597$
540 days	4.4%	$B_0^€(540) = 1/(1 + 0.044(540/360)) = 0.9381$
720 days	4.5%	$B_0^€(720) = 1/(1 + 0.045(720/360)) = 0.9174$

The fixed rate on a euro plain vanilla interest rate swap would be

$$R^€ = \left(\frac{360}{180}\right)\left(\frac{1-0.9174}{0.9814+0.9597\ 0.9381+0.9174}\right) = 0.0435.$$

million would be €10 million. Converting €10 million to dollars gives €10,000,000 ($0.9804) = $9,804,000, which is the dollar notional principal. Thus, these two interest rates, the exchange rate, and the two notional principals equate the present values of the two streams of payments at the start of the transaction.

If either set of payments is at a floating rate, we do not have to solve for a fixed rate and can be assured that the present value of the payments equals one unit of notional principal in the given currency. We know that because we know from studying interest rate swaps that the present value of the stream of floating payments is equivalent to the present value of the stream of fixed payments, provided the appropriate fixed rate is used.

Although the swap value is zero at the start, after the swap begins, its value will change. Valuation of the currency swap is obtained by finding the present value of the two streams of payments per unit of notional principal. This is precisely what we did when valuing an interest rate swap. We then adjust the value of the foreign stream of payments by the actual foreign notional principal and then convert it to the domestic currency using the new exchange rate. Let us illustrate this in Table 12.7, three months into the life of the swap with the new exchange rate being $0.9790. In addition to finding the present value of the fixed payments, as in this actual swap, we shall also determine the present value of the payments as if they were floating so that we can examine the valuation of the swap if it had been designed with floating payments.

The values computed in Table 12.7 are the values of the streams of payments per notional principal of either $1 or €1. Consider this the standard case. Now we must determine the values for the actual notional principals in the swap.

The value of the dollar fixed payments for a notional principal of $9.804 million is

$$\$9{,}804{,}000(1.01132335) = \$9{,}915{,}014.$$

If the payments had been floating, their value for a notional principal of $9.804 million would be

$$\$9{,}804{,}000(1.013115) = \$9{,}932{,}579.$$

The value of the euro fixed payments for a notional principal of €10 million is

$$€10{,}000{,}000(1.00883078) = €10{,}088{,}308.$$

If the euro payments had been floating, their value for a notional principal of €10 million would be

$$€10{,}000{,}000(1.0091157) = €10{,}091{,}157.$$

To determine the value of the currency swap, we must do two final things. We must first convert the values of the two streams of cash flows into a common currency and then net the two amounts. Typically we would prefer to convert to the home currency of the party from whose perspective we are valuing the swap. The actual swap was for Reston to pay euros fixed and receive dollars fixed. With an exchange rate of $0.9790, the value of the swap in dollars would be

$$\$9{,}915{,}014 - €10{,}088{,}308(\$0.9790/€) = \$38{,}560.$$

Since we have the necessary information, let us determine the values of the other possible swaps Reston could have arranged. The value if the swap had involved paying euros fixed and receiving dollars floating would be

$$\$9{,}932{,}579 - €10{,}088{,}308(\$0.9790/€) = \$56{,}125.$$

Table 12.7 Valuing a Currency Swap During Its Life

Three months into the Reston-GSI swap, the new term structures and zero coupon bond prices for dollars are

Term	Dollar rate	Discount Bond Price
90 days	5.7%	$B_{90}^{\$}(90) = 1/(1 + 0.057(90/360)) = 0.9860$
270 days	6.1%	$B_{90}^{\$}(270) = 1/(1 + 0.061(270/360)) = 0.9563$
450 days	6.4%	$B_{90}^{\$}(450) = 1/(1 + 0.064(450/360)) = 0.9259$
630 days	6.6%	$B_{90}^{\$}(630) = 1/(1 + 0.066(630/360)) = 0.8965$

The present value of the dollar fixed payments of 0.061(180/360) plus a $1 notional principal is

$$0.061\left(\frac{180}{360}\right)(0.9860 + 0.9563 + 0.9259 + 0.8965) + 1.0(0.8965) = 1.01132335$$

If the swap had been designed with floating payments, the present value of the dollar floating payments would be found by discounting the next floating payment, which is at the original 180-day floating rate of 5.5%, plus the market value of the floating-rate bond on the next payment date:

$$\left(1.0 + 0.055\left(\frac{180}{360}\right)\right)0.9860 = 1.013115$$

The new term structure and discount bond prices for the euro are

Term	Euro rate	Discount Bond Price
90 days	3.9%	$B_{90}^{€}(90) = 1/(1 + 0.039(90/360)) = 0.9903$
270 days	4.3%	$B_{90}^{€}(270) = 1/(1 + 0.043(270/360)) = 0.9688$
450 days	4.5%	$B_{90}^{€}(450) = 1/(1 + 0.045(450/360)) = 0.9467$
630 days	4.6%	$B_{90}^{€}(630) = 1/(1 + 0.046(630/360)) = 0.9255$

The present value of the euro fixed payments of 0.0435(180/360) plus a €1 notional principal is

$$0.0435\left(\frac{180}{360}\right)(0.9903 + 0.9688 + 0.9467 + 0.9255) + 1.0(0.9255) = 1.00883078$$

If the payments were floating, the present value of the euro payments would be found by discounting the next floating payment, which is at the original 180-day floating rate of 3.8%, plus the market value of the floating-rate bond on the next payment date:

$$\left(1.0 + 0.038\left(\frac{180}{360}\right)\right)0.9903 = 1.0091157.$$

The value if the swap had involved paying euros floating and receiving dollars fixed would be

$$\$9{,}915{,}014 - €10{,}091{,}157(\$0.9790/€) = \$35{,}771.$$

The value if the swap had involved paying euros floating and receiving dollars floating would be

$$\$9{,}932{,}579 - €10{,}091{,}157(\$0.9790/€) = \$53{,}336.$$

Note that the value of a currency swap is driven by changes in interest rates between the two countries and the change in the exchange rate. But regardless of the determinants, the value of a currency swap is simply the present value of one stream of payments minus the present value of the other stream of payments, accounting for the notional principals, and converting the payments to a common currency.

Currency Swap Strategies

We discussed the Reston-GSI swap without explaining why Reston might wish to enter into the swap. Now let us take a look at the possible motivation for Reston to do this transaction.

Reston Technology is an established Internet company in Northern Virginia's technology corridor. It is planning to expand its operations into Europe. To do so, it needs to borrow €10 million. It would like to issue bonds at a fixed rate and pay them back semiannually over two years. At the current exchange rate of $0.9804 per euro, Reston could borrow $9,804,000 and convert this amount to euros. Its expanded operations, however, will generate cash flow in euros so it would prefer to make its interest payments in euros.

While Reston considers borrowing in euros, its primary bank has a subsidiary, Global Swaps, Inc. (GSI), which is a large global derivatives dealer. GSI suggests that Reston borrow in U.S. dollars and engage in a currency swap to convert its loan to euros. Specifically Reston would borrow $9,804,000 in the U.S. market and enter into a currency swap with GSI in which GSI pays Reston 10 million up front and Reston pays GSI $9,804,000 up front. Of course, Reston would get this $9,804,000 from the loan it takes out in dollars. As we know from previously examining the swap, GSI will pay Reston interest in dollars at 6.1 percent semiannually for two years, and Reston will pay GSI interest at 4.35 percent in euros semiannually for two years. During this two-year period, Reston will make semiannual interest payments to its creditor at the rate it borrows at in dollars. After two years, GSI will pay Reston $9,804,000, which it will use to pay off its loan. Reston will pay GSI €10 million. The net effect is that Reston has issued a loan in dollars, but converted it to a loan in euros.

Reston is not likely to be able to borrow in dollars at the swap dollar fixed rate of 6.1 percent. That rate applies to high-quality London-based banks. Let us assume that Reston borrows at 6.5 percent. Thus, its loan interest payment will be $9,804,000 (0.065(180/360)) = $318,630. The overall transaction, consisting of the swap and the loan, is illustrated in Figure 12.8.

While Reston could have borrowed in euros, it would not likely have obtained terms as favorable as it gets by borrowing in dollars and using its relationship with GSI to save some money on the conversion of dollars to euros. GSI is a subsidiary of a large global bank, and can use its contacts, reputation, and expertise to operate on behalf of Reston in the international markets. GSI will hedge whatever risk it assumes.

Another reason why Reston might get better terms by borrowing in dollars and swapping into euros rather than borrowing in euros is that it assumes some credit risk resulting from the possibility that GSI might default. If GSI defaults, Reston would still have to make the dollar payments on its loan. If Reston borrowed in euros, it would have no credit risk, because it would not be receiving any payments from another party. For assuming this credit risk, Reston may be able to obtain a better rate.

Another possible use of a currency swap is in hedging a stream of foreign cash flows. Suppose that a firm expects to receive a stream of equivalent cash flows from its foreign operations. As we have discussed in previous chapters, it could use options, forwards, or futures to hedge the conversion of that stream into its domestic currency. But swaps can also be used. In this case, however, the firm would prefer a currency swap that does not involve the payment of notional principal.

For example, consider a firm called FXI with a Swiss subsidiary that generates annual cash flows of SF20 million, which are converted into dollars on the last day of the year. FXI would like to hedge the conversion of these cash flows into dollars for the next five years. In our previous treatment of currency swaps, we did not cover the pricing of currency swaps that do not involve the exchange of notional principals. This procedure is only slightly harder and simply requires the omission of the notional principal and the calculation of forward rates. Let us just assume that the fixed rate for such a swap is 5 percent for Swiss francs and 6 percent for dollars. The exchange rate is $0.65. The swap will consist of annual payments.

DERIVATIVES TOOLS

Concepts, Applications, and Extensions
Valuing a Currency Swap as a Series of Currency Forward Contracts

In Chapters 8 and 9, we learned about currency forward contracts, in which one party agrees to pay a certain amount of money in one currency at a future date, while the other agrees to pay a certain amount of money in another currency on that same date. Thus, the two parties have implicitly agreed to exchange a given amount of two currencies at a fixed rate at a future date. A currency swap is similar to a currency forward contract, but there are important differences. First, there is a *series* of exchanges of currency as opposed to a single exchange. Thus, a currency swap can be viewed as a combination of several currency forward contracts. Second, a currency forward contract is priced using the interest rates in the two countries and the exchange rate such that the contract will have zero value at the start. A currency swap is a series of payments in which the *overall* value is zero. Some of the component payments do not, however, have zero value.

Let us take a look at how a currency swap can be viewed as a series of currency forward contracts using the Reston-GSI currency swap we discuss in this chapter. Recall that the swap calls for Reston to pay euros to GSI and receive dollars from GSI. The notional principal is \$9,804,000 and €10,000,000, which is based on the exchange rate of \$0.9804. The term structures and associated information are repeated in the table below. Recall that we found the dollar rate as 6.1 percent and the euro rate as 4.35 percent. The payments were obtained as

$$\text{Dollar payments: } 9{,}804{,}000(0.061)\left(\frac{180}{360}\right) = 299{,}022.$$

$$\text{Dollar payments: } 10{,}000{,}000(0.0435)\left(\frac{180}{360}\right) = 217{,}500.$$

Term	Dollar rate	Discount Bond Price
180 days	5.5%	$B_0^{\$}(180) = 1/(1 + 0.055(180/360)) = 0.9732$
360 days	5.5%	$B_0^{\$}(360) = 1/(1 + 0.055(360/360)) = 0.9479$
540 days	6.2%	$B_0^{\$}(540) = 1/(1 + 0.062(540/360)) = 0.9149$
720 days	6.4%	$B_0^{\$}(720) = 1/(1 + 0.064(720/360)) = 0.8865$
Term	**Euro rate**	**Discount Bond Price**
180 days	3.8%	$B_0^{€}(180) = 1/(1 + 0.038(180/360)) = 0.9814$
360 days	4.2%	$B_0^{€}(360) = 1/(1 + 0.042(360/360)) = 0.9597$
540 days	4.4%	$B_0^{€}(540) = 1/(1 + 0.044(540/360)) = 0.9381$
720 days	4.5%	$B_0^{€}(720) = 1/(1 + 0.045(720/360)) = 0.9174$

Thus, the streams of payments for Reston are

- Four payments at 180, 360, 540, and 720 days:
  - Pay €217,500
  - Receive \$299,022
- One payment at 720 days:
  - Pay €10,000,000
  - Receive \$9,804,000

Let us treat these payments as forward contracts and find their market values. We shall, however, need to know the forward rates for the euro in terms of dollars. In Chapter 9 we learned how interest rate parity provides the forward rate based on the spot rate compounded at the domestic interest rate and discounted at the foreign interest rate. These forward rates are as follows:

$$\text{180-day forward rate: } \$0.9804\left(\frac{1+0.055(180/360)}{1+0.038(180/360)}\right)=\$0.9886.$$

$$\text{360-day forward rate: } \$0.9804\left(\frac{1+0.055(360/360)}{1+0.042(360/360)}\right)=\$0.9926.$$

$$\text{540-day forward rate: } \$0.9804\left(\frac{1+0.062(540/360)}{1+0.044(540/360)}\right)=\$1.0052.$$

$$\text{720-day forward rate: } \$0.9804\left(\frac{1+0.064(720/360)}{1+0.044(720/360)}\right)=\$1.0146.$$

Now we can find the market values of the forward contracts that are implicitly contained within the swap. The swap consists of either four or five payments, depending on how one views the last payment. Recall that the last payment is an interest payment and a principal payment. These could be combined, but here we shall treat them as separate payments.

The values of the implicit forward contracts contained within the swap are as follows:

First forward contract, expiring in 180 days:
($299,022 – €217,500($0.9886))0.9732 = $81,750.

Second forward contract, expiring in 360 days:
($299,022 – €217,500($0.9926))0.9479 = $78,800.

Third forward contract, expiring in 540 days:
($299,022 – €217,500($1.0052))0.9149 = $73,550.

Fourth forward contract, expiring in 720 days:
($299,022 – €217,500($1.0146))0.8865 = $69,455.

Fifth forward contract, expiring in 720 days:
($9,804,000 – €10,000,000($1.0146))0.8865 = –$303,183.

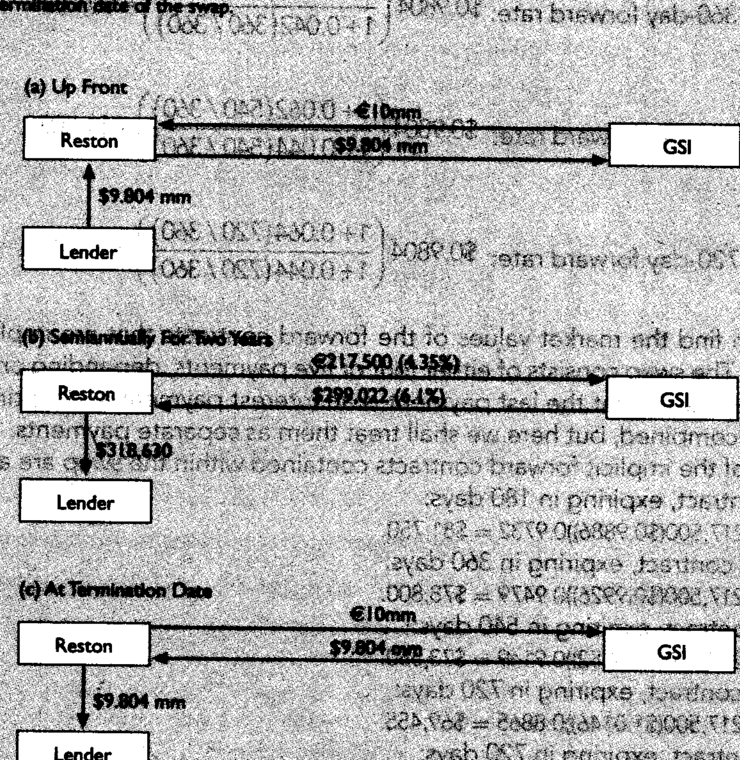
The sum of these values is $372, which is effectively zero (only 0.004% of the notional principal) but is not precisely zero only because of rounding many of these input values. Thus, the swap consists of five implicit forward contracts, the first four of which have positive value and the last of which has negative value that offsets and effectively makes the overall transaction have a value of zero. Ordinarily if a party constructed five currency forwards, the rates on these contracts would be set individually according to the term structure and the forward exchange rates. In a currency swap, the implicit forward contracts are treated as a package with the payments made at the same rate. A currency swap is more efficient than a group of individual currency forward contracts, because it combines into a single transaction what would otherwise take five transactions.

To generate an annual swap payment of SF20 million if the fixed rate is 5 percent would require a notional principal of SF20,000,000/(0.05) = SF400 million. This notional principal converts to a dollar notional principal of SF400,000,000($0.65) = $260 million. The dollar swap payments would, therefore, be $260,000,000(0.06) = $15,600,000.

Thus, FXI enters into a swap with notional principal of SF400 million and $260 million. The swap will require FXI to pay the counterparty SF400,000,000(0.05) = SF20 million, which will come from its subsidiary's

FIGURE 12.8 Example of a Currency Swap

Scenario: Reston issues a loan of $9.804 million at 6.5 percent with interest paid semiannually for two years. Reston prefers to borrow in euros so it enters into a currency swap with GSI for €10 million and $9.804 million. Payments are made semiannually at 6.1 percent in dollars and 4.35 percent in euros. Principal payments are exchanged up front and at the termination date of the swap.



cash flows. The counterparty will pay FXI $260,000,000(0.06) = $15,600,000. Thus, FXI has locked in the conversion of its Swiss franc cash flows to dollars. Of course, FXI bears the risk that its cash flows will deviate from SF20 million, but of course, it would face that risk whether it uses swaps, options, forwards, or futures.

The third type of swap we study in this chapter is the equity swap.

EQUITY SWAPS

In an equity swap at least one of the two streams of cash flows is determined by a stock price, the value of a stock portfolio, or the level of a stock index. The other stream of cash flows can be a fixed rate, a floating rate such as LIBOR, or it can be determined by the value of another stock, stock portfolio, or stock index. In this manner, an equity swap can substitute for trading in an individual stock, stock portfolio, or stock index. In this section we shall just refer to the underlying equity as a stock, which could represent an individual stock, a stock portfolio, or a stock index.

Equity swaps are certainly similar to interest rate and currency swaps, but they also differ notably. One difference is that the swap payment is determined by the return on the stock. Since stock returns can be negative, the swap payment can be negative. That is, suppose party A agrees to pay party B the return on the underlying stock. Suppose that at a given payment date, the return on the stock is negative. Then party A effectively owes a negative return. This means that party B would have to pay the return to party A. Unless party B also owes a negative return, party B will end up making both payments.

Another way in which equity swaps differ from interest rate and currency swaps is the fact that the upcoming equity payment is never known. The upcoming floating payment in an interest rate or currency swap is always known, and of course, a fixed payment would always be known. In an equity swap, however, the equity return is not determined until the end of the settlement period, which of course is when the payment is due.

Finally we should note that there is no adjustment such as days/360 for the equity payment on an equity swap.[11] It is determined strictly as the return on the stock over a given period, and no such adjustment is required.

Structure of a Typical Equity Swap

For an equity swap in which a party pays the return on the stock and receives a fixed rate, the cash flow will be

(Notional Principal) ((Fixed rate)q – Return on stock over settlement period).

Consider an investment management company called IVM. It wants to enter into an equity swap to pay the return on the Standard & Poor's 500 Total Return Index and receive a fixed rate. This index includes the effect of reinvesting dividends.[12] On the day the swap is arranged, the index is at 2,710.55. The swap will call for payments every 90 days for a 360-day period. Financial Swaps (FNS), the dealer, offers IVM a fixed rate of 3.45 percent with payments calculated on the basis of 90 days divided by 360. The notional principal will be $25 million. Let us treat the payment dates as day 90, day 180, day 270, and day 360. The initial day is day 0. Let S_0, S_{90}, etc. be the S&P 500 Total Return Index on day 0, day 90, and so forth. The cash flow to IVM on the settlement date will be

$$\$25{,}000{,}000\left(0.0345\left(\frac{90}{360}\right) - \text{Return on stock over settlement period}\right).$$

Obviously the fixed component of the payment will be

$$\$25{,}000{,}000\,(0.0345)\left(\frac{90}{360}\right) = \$215{,}625.$$

Table 12.8 contains an example of the payments that might occur on such a swap. Keep in mind that these are hypothetical results, based on assumptions about the course of the S&P 500 Total Return Index. The parties to the swap do not know what these payments will be when the swap is initiated.

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[11]Of course, if the other side of the transaction makes a fixed or floating interest payment, the (days/360 or 365) adjustment is required.

[12]The concept of a total return means that it includes capital gains and dividends. Most standard stock indices reflect only the prices of the component stocks, and, thus, incorporate only capital gains.

Table 12.8 After-the-Fact Payments in Equity Swap to Pay S&P 500 Total Return Index and Receive a Fixed Rate of 3.45 Percent

Notional Principal: $25,000,000

Day	Fixed Interest Payment	S&P Total Return Index	S&P Payment	Net Payment
0		2,710.55		
90	$215,625	2,764.90	$501,282	–$285,657
180	215,625	2,653.65	–1,005,913	1,221,538
270	215,625	2,805.20	1,427,750	–1,212,125
360	215,625	2,705.95	–864,518	1,100,143

Note: This combination of outcomes on the above dates represents only one of an infinite number of possible outcomes to the swap. They are used only to illustrate how the payments are determined and not the likely results.

Let us determine the first equity payment. When the swap is initiated on day 0, the S&P 500 Total Return Index is at 2,710.55. On day 90, the index is at 2,764.90. The equity payment would, therefore, be

$$\$25{,}000{,}000\left(\frac{2{,}764.90}{2{,}710.55}-1\right)=\$501{,}282.$$

In other words, the rate of return on the Index is (2,764.90/2,710.55) – 1 = 0.02005, or about 2 percent. Multiplied by $25,000,000 (retaining full decimal accuracy) gives a payment of $501,282. Thus, for the first payment IVM owes $501,282 and is due $215,625, for a net payment of –$285,657. So IVM pays the dealer $285,657.

For the second payment, IVM still receives $215,625. Note that the S&P 500 Total Return Index fell to 2,653.65, a return of (2,653.65/2,764.90) – 1 = –0.0402 or –4.02 percent, which amounts to $1,005,913. Now, IVM owes a negative return on the stock, which means that the counterparty pays IVM. So, IVM is due a cash flow of $215,625 – (–$1,005,913) = $1,221,538.

Of course, IVM could have constructed the swap so that it received a floating payment based on LIBOR. In that case, the calculations would be similar to those above, except that LIBOR on day 0 would determine the interest payment on day 90, LIBOR on day 90 would determine the interest payment on day 180, etc. The payoff formula would be

[redacted]

Alternatively, IVM could have constructed the swap to involve payment of the return on the S&P 500 Total Return Index and receipt of the return on some other stock index.

[redacted]

Suppose for example, that IVM wanted to receive the return on the NASDAQ Index. Then the interest payment it receives in the example above would be replaced by the return on the NASDAQ Index, which would be computed in the same manner as the return on the S&P 500 Total Return Index. When we examine equity swap strategies, we shall take a look at why these different types of swaps would be used.

Pricing and Valuation of Equity Swaps

We shall now look at the pricing and valuation of the three types of equity swaps: a swap involving an equity return vs. a fixed rate, a swap involving an equity return vs. a floating rate, and a swap involving one equity return vs. another equity return.

We start with the swap to pay a fixed rate and receive the equity return. The fixed rate is denoted as R, and the notional principal is \$1. To determine the value of the swap, we need to construct a strategy that will replicate the payments on the equity swap. We can do this quite easily.

- Invest \$1 in the stock.
- Issue a \$1 face value loan with an interest rate of R. Pay interest on each of the swap settlement dates and repay the principal at the swap termination date. Interest payments will be calculated on the basis of days/360.

We shall use the symbol q for (days/360).

Let us now see why this strategy works. Suppose we execute the above transactions for the purpose of replicating a one-year swap with semiannual payments on days 180 and 360. Then, on day 180, we would have stock worth S_{180}/S_0, and we would owe Rq.

Suppose we sell the stock and withdraw only the return, $S_{180}/S_0 - 1$, which can be positive or negative. If the return is positive, we have a cash inflow. If it is negative, we have a cash outflow. Combined with the interest payment, the overall net cash flow is $S_{180}/S_0 - 1 - Rq$. This is precisely the payment on the swap. Because we withdrew only the return from the stock sale, we have \$1 left over, which we reinvest in the stock. Now let us proceed forward to the next period.

On day 360, the stock would be worth S_{360}/S_{180}. We pay back the loan principal of \$1 and the interest of Rq. We liquidate the stock. Then the net cash flow would be $S_{360}/S_{180} - 1 - Rq$. This is precisely the cash flow on the swap. Thus, this strategy replicates the equity swap. In general, for a swap with n payments, the strategy will cost \$1 to buy the stock, which is offset by the cash flow from the loan. The overall cost to establish the position is the value of the position when it is established.

$$1 - B_0(t_n) - R_q \sum_{i=1}^{n} B_0(t_i).$$

The first term, 1, is the \$1 invested in the stock. The second term, $-B_0(t_n)$, is the loan principal due on day t_n. The term with the summation is the present value of the series of loan interest payments of Rq on each swap payment date.

Since the swap is established so that its value at the start is zero, we set the above equation to zero and solve for R:

$$R = \left(\frac{1}{q}\right)\left(\frac{1 - B_0(t_n)}{\sum_{i=1}^{n} B_0(t_i)}\right)$$

Interestingly, this rate is the same rate as the fixed rate on a plain vanilla swap. This should make sense. The swap is designed to equate the present value of the equity payments with the present value of the fixed interest payments. Since we start off by investing \$1 in the stock, the present value of the equity payments is \$1. To make the present value of the fixed interest payments equal \$1, we need only set the payments at the fixed rate on a plain vanilla swap.

Table 12.9 shows how the IVM swap is valued.

Now let us consider a time during the life of the swap at which we wish to determine the value of the swap. Assume that this is before the first payment date. The problem will be a little simpler if we look at it from the standpoint of the party receiving the equity return and paying the fixed rate. On the first payment day, we shall receive an equity payment of $S_{90}/S_0 - 1$. We could replicate this payment by purchasing $1/S_0$ shares, currently at S_t, which will cost $(1/S_0)S_t$. At the next payment date, we will have stock worth $(1/S_0)S_{90}$.[13] We sell this stock, generating a cash flow of $(1/S_0)S_t - 1$, which could be positive or negative, plus the \$1 left over, which we reinvest into the stock. This will replicate the stock payment the next period. This procedure continues throughout the life of the swap and will leave \$1 at the end. Thus, to replicate the cash flows, we do the following:

- Invest $(1/S_0)S_t$, which equals S_t/S_0, in the stock. Liquidate and reinvest as described.
- Issue a \$1 loan at an interest rate of R, with interest to be paid on each of the swap settlement dates and the principal to be repaid at the swap termination date. Interest payments will be calculated on the basis of $q = \text{days}/360$.

Table 12.9 Pricing an Equity Swap

IVM enters into an equity swap to receive a fixed rate and pay the return on the S&P 500 Total Return Index. The payments will be quarterly for one year with interest calculated using the adjustment factor 90/360. The term structure is as follows:

Term	Rate	Discount Bond Price
90 days	$L_0(30) = 3\%$	$B_0(30) = 1/(1 + 0.03(90/360)) = 0.9926$
180 days	$L_0(120) = 3.2\%$	$B_0(120) = 1/(1 + 0.032(180/360)) = 0.9843$
270 days	$L_0(270) = 3.3\%$	$B_0(270) = 1/(1 + 0.033(270/360)) = 0.9758$
360 days	$L_0(300) = 3.5\%$	$B_0(300) = 1/(1 + 0.035(360/360)) = 0.9662$

With $q = 90/360$, then $1/q = 360/90$, and the fixed rate would, therefore, be

$$R = \left(\frac{360}{90}\right)\left(\frac{1-0.9662}{0.9926+0.9843+0.9758+0.9662}\right) = 0.0345$$

Thus, the rate would be 3.45 percent.

This strategy will replicate the payments on the swap. The cost to establish this strategy is

$$\left(\frac{S_t}{S_0}\right) - B_t(t_n) - Rq\sum_{i=1}^{n} B_t(t_i).$$

The first term is the investment in stock necessary to replicate the equity cash flows. The second is the present value of the repayment of the loan principal at the swap termination date, and third is the present value of the set of payments of Rq on each of the swap settlement dates.

Let us apply the formula to our example of the IVM swap. Suppose it is 60 days into the life of the swap. Table 12.10 presents a set of new interest rates and the value of the equity swap at this time.

Now suppose we are interested in an equity swap with floating payments. We know that at the start the present value of the fixed payments (including a notional principal of 1) at the rate R is 1 and that this equals the present value if the payments were floating (including the notional principal). Thus, the value of a swap involving the payment of the equity return against floating payments is zero at the start, as it should be. To value the swap during its life, notice that we can replicate the equity swap involving floating payments by doing the following:

- Enter into an equity swap to pay the equity return and receive a fixed rate
- Enter into a plain vanilla swap to pay a fixed rate and receive a floating rate

The fixed payments would cancel, and this would net out to an equity swap to pay the equity return and receive a floating rate. We have already valued the equity swap to pay the equity return and receive a fixed rate and obtained a value of –\$227,964. We now need only value a plain vanilla swap to pay a fixed rate and

[13]This expression is $1/S_0$ shares worth S_{90} per share.

Table 12.10 Valuing an Equity Swap During its Life

We are now 60 days into the life of the IVM swap. The new term structure is as follows:

Term	Rate	Discount Bond Price
30 days	$L_{60}(30) = 3.50\%$	$B_{60}(30) = 1/(1 + 0.035(30/360)) = 0.9971$
120 days	$L_{60}(120) = 3.75\%$	$B_{60}(120) = 1/(1 + 0.0375(120/360)) = 0.9877$
210 days	$L_{60}(210) = 3.90\%$	$B_{60}(210) = 1/(1 + 0.039(210/360)) = 0.9778$
300 days	$L_{60}(300) = 4.00\%$	$B_{60}(300) = 1/(1 + 0.04(300/360)) = 0.9677$

The stock index is at 2,739.60. Thus, the value of the swap per $1 notional principal is

$$0.0345\left(\frac{2{,}739.60}{2{,}710.55}\right) - 0.9677 - 0.0345\left(\frac{90}{360}\right)(0.9971 + 0.9877 + 0.9778 + 0.9677) = 0.00911854.$$

This formulation, however, is from the perspective of the party paying the fixed rate and receiving the equity return. So to IVM, the value is actually 20.00911854 per $1 notional principal. Thus, for a notional principal of $25 million, the value of the swap is

$$\$25{,}000{,}000(-0.00911854) = -\$227{,}964.$$

receive the floating rate. Recall that we learned this earlier in the chapter. Using the information in Table 12.10, the value of the fixed payments, plus hypothetical notional principal, is

$$0.0345\left(\frac{90}{360}\right)(0.9971 + 0.9877 + 0.9778 + 0.9677) + 1.0(0.9677) = 1.00159884.$$

Recall from our examination of interest rate swaps that we value the floating payments as the present value of the upcoming floating payment plus the par value of 1. The upcoming floating payment is at 3 percent. Thus, the value of the floating payments plus hypothetical notional principal is

$$\left(1 + 0.03\left(\frac{90}{360}\right)\right)0.9971 = 1.00457825.$$

Thus, the value of a plain vanilla swap to pay fixed and receive floating is

$$1.00457825 - 1.00159884 = 0.00297941.$$

For a $25 million notional principal, this value is

$$\$25{,}000{,}000(0.00297941) = \$74{,}485.$$

So the value of the equity swap to pay the equity return and receive a floating payment is

$$-\$227{,}964 + \$74{,}485 = -\$153{,}479.$$

If the swap involves the payment of one equity return for another, we must develop a new strategy for replicating the payments. Let $S_0(1)$ be the value of stock index 1 on day 0 and $S_0(2)$ be the value of stock index 2 on day 0. We then change the subscript to reflect later days in the life of the swap. Assume that we pay the return on stock index 2 and receive the return on stock index 1 with payments on days 180, 360, etc. The first cash flow on the swap will be $(S_{180}(1)/S_0(1)) - 1 - (S_{180}(2)/S_0(2)) - 1) = S_{180}(1)/S_0(1) - S_{180}(2)/S_0(2)$. The remaining cash flows will be done in a similar manner, changing the time subscripts to 360 vs. 180 and so forth.

To replicate the swap, we sell short $1 of stock index 2, take the proceeds, and buy $1 of stock index 1. This will require no money of our own. At time 1 we sell stock index 1, generating cash of $S_{180}(1)/S_0(1)$. We withdraw the cash return, $(S_{180}(1)/S_0(1)) - 1$, and reinvest $1 in stock index 1. We cover the short sale of stock index 2 by buying back the stock. This will require an outlay of $S_{180}(2)/S_0(2)$. We then sell short $1 of stock index 2 so our net flow from the transaction in stock index 2 is $-(S_{180}(2)/S_0(2) - 1)$. The overall net cash flow is $S_{180}(1)/S_0(1) - S_{180}(2)/S_0(2)$, which is the cash flow from the swap. We now move forward with $1 invested in stock index 1 and $1 sold short of stock index 2. If we proceed in this manner, we will generate the cash flows on the swap. Since there is no cost to establishing the position, the value of the swap is zero at the start, as it should be.

It is very simple to value this type of swap during its life. We value it as a position of being long in one stock and short the other. Suppose in the IVM example, we change the fixed payment that IVM will receive to be the return on the NASDAQ stock index. Let the index be 1,835.24 at the start of the transaction. Sixty days later, the index is at 1,915.71. Then the value of the swap is easily found as

$$\left(\frac{1{,}915.71}{1{,}835.24}\right) - \left(\frac{2{,}739.60}{2{,}710.55}\right) = 0.03312974.$$

For a $25 million notional principal swap, the value is

$$\$25{,}000{,}000(0.03312974) = \$828{,}244.$$

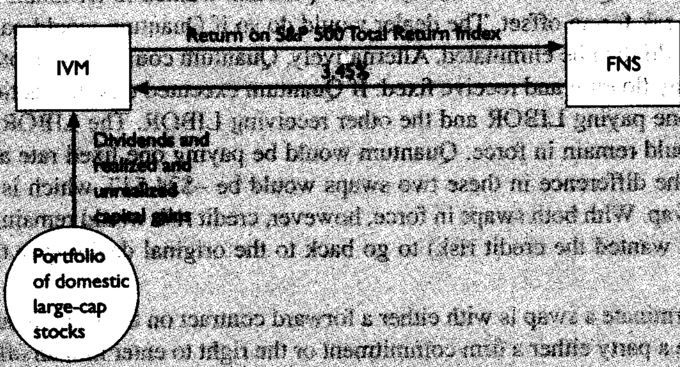
Equity Swap Strategies

Equity swaps are useful strategies for equity investors. An equity swap to pay a fixed rate and receive the equity return is essentially equivalent to issuing a fixed-rate bond and using the proceeds to buy stock. An equity swap to pay a floating rate and receive the equity return is essentially the same as issuing a floating-rate bond and using the proceeds to buy stock. An equity swap to pay the return on one stock index and receive the return on another is essentially the same as selling short one stock and using the proceeds to buy another. Of course, certain transactions are required at the payment dates, so these swap strategies are not identical to buying and holding stock. In addition, the payments required on equity swaps make them a form of buying stocks using leverage. Nonetheless, equity swaps are very similar to stock transactions and can serve as valuable substitutes for stock transactions, as is the case for virtually any equity derivative.

Going back to the IVM strategy, recall that IVM is an asset management company. It holds a portfolio of stock. In some cases it needs to make adjustments to its portfolio. These adjustments typically involve buying and selling stock. As we have seen throughout this book, assets can be bought and sold synthetically using derivatives. In this case, a company like IVM could use equity swap to synthetically replicate the sale of stock and purchase of another asset. Suppose IVM wants to sell some domestic large-cap stock and buy a fixed-rate bond. Then it might enter into an equity swap to pay the return on an index of domestic large-cap stock, such as the S&P 500 Total Return Index, and receive a fixed return. The swap we described earlier is precisely this type of transaction. A diagram of the overall transaction is shown in Figure 12.9. In doing the transaction, IVM effectively sells domestic large-cap stock, as represented by the S&P 500 Total Return Index, and converts to a fixed rate bond paying 3.45 percent.

You should be aware, however, that IVM assumes some risk in this transaction. Besides the aforementioned fact that the swap dealer could default, there are two other important risks. One is that the performance of the domestic, large-cap stock portfolio will not precisely match the performance of the S&P 500 Total Return Index. This is a form of basis risk, similar to the risk we discussed in using futures. In the investment management field,

FIGURE 12.9 Equity Swap of IVM to Exchange the Return on Domestic Large-Cap Stock for a Fixed Return



this risk is usually referred to as tracking error, meaning the risk that the index on which the derivative is based will not track the underlying portfolio. Obviously this risk is not new to us. It applies anytime a derivative on an index is used with a portfolio that does not precisely match the index.

Another risk is that if the swap generates net cash outflows, the firm must produce the cash to make the swap payments. While the portfolio could be earning a sufficiently high return, that return is likely to include some unrealized capital gains. If there is not sufficient cash to make the swap payment, the firm could be forced to sell some stock to generate the cash. This would then defeat the purpose of using the swap, which was to avoid selling the stock in the first place. Thus, to use an equity swap, a firm would need to take into account the potential for cash outflows and would need to set aside a liquidity pool.

As we have noted, variations of this strategy include having the interest payment be at a floating rate. In that case, the strategy would be approximately equivalent to selling stock and investing the proceeds in a floating-rate bond. If the firm wanted to sell the stock and buy a floating-rate bond, this type of equity swap would be a good substitute.

Alternatively, if the firm did a swap in which it pays the return on the S&P 500 and the dealer pays it the return on another index, such as the NASDAQ index, the firm would be synthetically selling stock as represented by the S&P 500 and buying stock as represented by the NASDAQ index.

SOME FINAL WORDS ABOUT SWAPS

We began the chapter by mentioning that swaps are like combinations of forward contracts. This is an important analogy, but it applies only to swaps that involve fixed payments, as in forward contracts. For swaps with strictly floating payments, the analogy breaks down. Moreover, for swaps with fixed payments, the analogy is only partial. For example, if a party engaged in a series of currency forward contracts, the forward rate would be different for each expiration. After all, forward prices and rates nearly always differ by maturity. A currency swap with fixed payments would involve payments at the same rate.

Because swaps are similar to forward contracts, swaps will also be similar to futures contracts. In addition, swaps can be shown to be similar to combinations of options. In the next chapter, we shall look at interest rate options and show how they can be combined to replicate interest rate swaps.

Swaps are nearly always designed with the intention of holding the position until the termination date. In some cases, a party changes its mind and wants out of the swap. There are a number of ways it can exit a swap. The primary way is to go back to the dealer and ask to terminate the swap or enter an offsetting swap.

In effect, the parties would do the opposite swap and cancel the original swap. For example, recall the Quantum Electronics plain vanilla swap in which it paid a fixed rate and received LIBOR. Ninety days into its life, the market value to Quantum was –$24,537. If Quantum wanted to terminate the swap, it could go back to the dealer and ask for an offset. The dealer would do so if Quantum would pay the market value of $24,537. The swap would then be eliminated. Alternatively, Quantum could go to another dealer and ask for a quote on a swap to pay floating and receive fixed. If Quantum executed this transaction, it would have two swaps in place, with one paying LIBOR and the other receiving LIBOR. The LIBOR effects would offset, though both swaps would remain in force. Quantum would be paying one fixed rate and receiving another. The present value of the difference in these two swaps would be –$24,537, which is precisely the market value of the original swap. With both swaps in force, however, credit risk would remain, and Quantum would be better off (unless it wanted the credit risk) to go back to the original dealer, pay the market value, and terminate the swap.

Another way to terminate a swap is with either a forward contract on the swap or an option on the swap. These instruments give a party either a firm commitment or the right to enter into an offsetting swap at terms agreed to in advance. These instruments, called forward swaps and swaptions, will be covered in Chapter 13.

Questions and Problems

1. Consider a $100 million equity swap with semiannual payments. When the swap is established, the underlying stock is at 1,215.52. One party pays a fixed rate of 5.5 percent based on the assumption of 30 days per month and 360 days in a year. If the stock index is at 1,275.89 on the first payment date, calculate the net swap payment, indicating which party pays it.
2. Consider a currency swap for $10 million and SF15 million. One party pays dollars at a fixed rate of 9 percent, and the other pays Swiss francs at a fixed rate of 8 percent. The payments are made semiannually based on the exact day count and 360 days in a year. The current period has 181 days. Calculate the next payment each party makes.
3. The CEO of a large corporation holds a position of 25 million shares in her company's stock, which is currently priced at $20 and pays no dividends. She is concerned that, because of her large shareholdings and the fact that her compensation is tied to the performance of the stock, she is very poorly diversified. She does not think it is wise to sell a significant amount of stock, because she knows that she needs to be heavily invested in the stock to satisfy the shareholders, and she values the voting rights she has from owning so many shares. Nonetheless, she would be interested in synthetically selling about five million shares using an equity swap. Assume the role of a swap dealer and present three possible equity swap proposals, which are based on the three different types of cash flows that could be paid against payment of the return on the stock.
4. A corporation enters into a $35 million notional principal interest rate swap. The swap calls for the corporation to pay a fixed rate and receive a floating rate of LIBOR. The payments will be made every 90 days for one year and will be based on the adjustment factor 90/360. The term structure of LIBOR when the swap is initiated is as follows:

Days	Rate
90	7.00%
180	7.25
270	7.45
360	7.55

   a. Determine the fixed rate on the swap.
   b. Calculate the first net payment on the swap.

c. Assume that it is now 30 days into the life of the swap. The new term structure of LIBOR is as follows:

Days	Rate
60	6.80%
150	7.05
240	7.15
330	7.20

Calculate the value of the swap.

5. Show how to combine a currency swap paying Swiss francs at a floating rate and receiving Japanese yen at a floating rate with another currency swap to obtain a plain vanilla swap paying Swiss francs at a floating rate and receiving Swiss francs at a fixed rate.
6. A bank currently holds a loan with a principal of $12 million. The loan generates quarterly interest payments at a rate of LIBOR plus 300 basis points, with the payments made on the 15th of February, May, August, and November on the basis of the actual day count divided by 360. The bank has begun to believe that interest rates will fall. It would like to use a swap to synthetically alter the payments on the loan it holds. The rate it could obtain on a plain vanilla swap is 7.25 percent. Explain how the bank would use a swap to achieve this objective.
7. Explain why interest rate swaps are more widely used than currency and equity swaps.
8. A U.S. corporation is considering entering into a currency swap that will call for the firm to pay dollars and receive British pounds. The dollar notional principal will be $35 million. The swap will call for semiannual payments using the adjustment 180/360. The exchange rate is $1.60. The term structures of dollar LIBOR and pound LIBOR are as follows:

Days	Dollar LIBOR	Pound LIBOR
180	7.00%	6.50%
360	7.25	7.10
540	7.45	7.50
720	7.55	8.00

Answer the following questions.

a. Determine the appropriate pound notional principal. Use this result in each of the remaining questions.
b. Determine the fixed rates in dollars and in pounds.
c. For each of the following cases, determine the first payment on the swap:
   i. Dollars fixed, pounds fixed
   ii. Dollars fixed, pounds floating
   iii. Dollars floating, pounds floating
   iv. Dollars floating, pounds fixed
d. Now assume it is 120 days into the life of the swap. The new exchange rate is $1.42. The new term structures are as follows:

Days	Dollar LIBOR	Pound LIBOR
60	6.80%	6.40%
240	7.05	6.90
420	7.15	7.30
600	7.20	7.45

Determine the value of the swap for each of the following cases:

i. Dollars fixed, pounds fixed
ii. Dollars fixed, pounds floating
iii. Dollars floating, pounds floating
iv. Dollars floating, pounds fixed

9. A pension fund wants to enter into a six-month equity swap with a notional principal of $60 million. Payments will occur in 90 and 180 days. The swap will allow the fund to receive the return on a stock index, currently at 5,514.67. The fund is considering three different types of swaps, one of which would require it to pay a fixed rate, another that would require it to pay floating rate, and another that would require it to pay the return on another stock index, which is currently at 1,212.98. Refer to these as swaps 1, 2, and 3. The term structure is as follows:

Term	Rate	Discount Bond Price
90 days	9%	$B0(90) = 1/(1 + 0.09(90/360)) = 0.9780$
180 days	10	$B0(180) = 1/(1 + 0.10(180/360)) = 0.9524$

a. Find the fixed rate for swap 1.
b. Find the payments on day 90 for swaps 1, 2, and 3. For swap 3, assume that on day 90 stock index 1 is at 5,609.81 and stock index 2 is at 1,231.94. Be sure to indicate the net payment.
c. Assume it is 30 days into the life of the swap. Stock index 1 is at 5,499.62, and stock index 2 is at 1,201.45. The new term structure is as follows:

Term	Rate	Discount Bond Price
60	6.80%	$B30(90) = 1/(1 + 0.068(60/360)) = 0.9888$
150	7.05	$B30(180) = 1/(1 + 0.0705(150/360)) = 0.9715$

Find the values of swaps 1, 2, and 3..

10. You are a pension fund manager who anticipates having to pay out 8 percent (paid semiannually) on $100 million for the next seven years. You currently hold $100 million of a floating-rate note that pays LIBOR + 2 1/2 percent. You view this as an attractive investment but realize that if LIBOR falls below 5 1/2 percent, you will not have enough cash to make your fixed payments. You arrange a swap with a dealer who agrees to pay you 6 percent fixed, while you pay it LIBOR. Determine your cash flow as a percent of the notional principal at each payment date under this arrangement. Assume for simplicity that each period is 180 days and that there are 360 days in the year.
11. A hedge fund is currently engaged in a plain vanilla euro swap in which it pays euros at the euro floating rate of Euribor and receives euros fixed. It would like to convert this position into one in which it pays the return on the S&P 500 and receives euros at a fixed rate. Show how it can use currency and equity swaps to maintain its position in the plain vanilla euro swap and convert its overall position to the one desired.
12. (Concept Problem) An asset management firm has a $300 million portfolio consisting of all stock. It would like to divest 10 percent of its stock and invest in bonds. It considers the possibility of synthetically selling some stock using equity swaps. It does not, however, want to receive a fixed or floating rate. If it actually sold the stock, it would invest in a broadly diversified portfolio of bonds. In fact, there are bond indices that are quite representative of the universe of bonds in which it would invest. Design a strategy using swaps that would enable it to achieve its objective.
13. Why is notional principal often exchanged in a currency swap but not in an interest rate or equity swap? Why would the parties to a currency swap choose not to exchange the notional principal?
14. Consider a $30 million notional principal interest rate swap with a fixed rate of 7 percent, paid quarterly on the basis of 90 days in the quarter and 360 days in the year. The first floating payment is set at 7.2 percent. Calculate the first net payment and identify which party, the party paying fixed or the party paying floating, pays.

15. Explain how the following types of swaps are analogous to transactions in bonds.
    a. Interest rate swaps
    b. Currency swaps
16. A swap dealer quotes that the rate on a plain vanilla swap, for it to pay fixed, is the five-year Treasury rate plus 10. To receive fixed, the dealer quotes the rate as the five-year Treasury rate plus 15. Assuming the five-year Treasury rate is 7.60 percent, explain what these quotes mean.
17. The U.K. manager of an international bond portfolio would like to synthetically sell a large position in a French government bond, denominated in euros. The bond is selling at its par value of €46.15 million, which is equivalent to £30 million at the current exchange rate of £0.65. The bond pays interest at a fixed rate of 5.2 percent annually for 10 years. The manager would like to sell the bond and invest the proceeds in a pound-denominated floating-rate bond. Design a currency swap strategy that would achieve the desired objective and identify the payments that would occur on the overall position, which includes both the French bond and the swap. The fixed rates on the currency swap are 4.9 percent in pounds and 5.7 percent in euros.
18. Explain how swaps are similar to but different from forward contracts.
19. Suppose that a party engages in a swap, but before the expiration date of the swap, the party decides that it would like to terminate the position. Explain how it can do so.
20. Explain how an interest rate swap is a special case of a currency swap.
21. (Concept Problem) Consider a currency swap with but two payment dates, which are one year apart, and no exchange of notional principals. On the first date, the party pays U.S. dollars at a rate of 4 percent and receives British pounds at a rate of 3.5 percent. Since the payments are annual, no adjustment, such as days/360, is necessary. The notional principals are $10 million and £6.25 million. Explain from an American's perspective how this transaction is like a series of forward contracts on the pound. Also, explain how the transaction can be fairly priced, which you can assume it is, even though the implied forward rate is the same for both maturities.

PART 3

Advanced Topics

13

INTEREST RATE FORWARDS AND OPTIONS

While this chapter deals with certain Interest Rate Derivatives, we have already covered the most widely used interest rate derivative, the plain vanilla interest rate swap. In addition, we have already covered Eurodollar futures, which are also appropriately called interest rate derivatives. In some cases, futures on such instruments as Treasury bonds and notes are also called interest rate derivatives. But we need to make an important distinction between derivative contracts on interest rates and derivative contracts on fixed-income securities.

Consider a Eurodollar time deposit that pays \$1 in 90 days. At a current rate of 8 percent, this Eurodollar has a value of $\$1/(1 + 0.08(90/360)) = \$0.9804$. Suppose that a forward contract had been created on this Eurodollar and that contract is expiring right now. The payoff of a long position would be \$0.9804 minus the forward price agreed to when the contract was initiated. The payoff of a forward contract on the Eurodollar interest rate, LIBOR, would be 0.08(90/360) minus the forward rate agreed to when the contract was initiated. Forward contracts on interest rates and forward contracts on Eurodollars are related but different contracts.

Recall that Eurodollars are priced based on the add-on interest method. Hence, we found the price of the Eurodollar above as $\$1/(1 + 0.08(90/360))$. Treasury bills are based on the discount interest method. For a rate of 8 percent, a Treasury bill price would be $\$1(1 - 0.08(90/360)) = \$0.98$. A long position in a forward contract on a Treasury bill would pay off 1 minus 0.08(90/360) minus the forward price. A short position in a forward contract on the rate would pay off the forward rate minus 0.08 times 90/360. It should be easy to see that a long position in a forward contract on a discount instrument is equal to a short position in a forward contract on a rate with the forward rate based on one minus the forward price. Similar comments can be made for options.

Thus, in some cases, derivatives on interest rates are essentially the same as derivatives on bonds. For others, this is not the case. In this chapter we shall focus on derivative on interest rates. In some cases we shall gain some advantage by noting the equivalence with derivatives on bonds. But in most cases, we shall just focus on the fact that the underlying is an interest rate and that its payoff formula is based directly on the underlying interest rate rather than the price of a bond.

This chapter is called "Interest Rate Forwards and Options" and will cover four primary types of instruments. The first is the interest rate forward, more commonly known as a forward rate agreement, or FRA. This instrument is a forward contract in which the two parties agree to make interest payments to each other at futures dates. One party makes a payment at a rate agreed to in advance. The other party makes a payment at a rate to be determined later. The second type of instrument is the interest rate option, in which one party pays the other a premium today and receives the right to either make a known interest payment and receive an unknown interest payment at a future date or receive a known interest payment and make an unknown interest payment at a future date. The right to make a known payment is an interest rate call. The right to receive a known payment is an interest rate put.

In addition to standard interest rate forwards and options, we shall cover two other types of forward and option contracts involving interest rates. The third instrument we cover in this chapter is an option to enter into a swap, which is commonly referred to as a swap option, or swaption. The buyer of a swaption in which the underlying is an interest rate swap pays a premium and receives the right to enter into a swap to pay the fixed rate and receive the floating rate or pay the floating rate and receive the fixed rate. These instruments may seem like calls and puts, but these terms are not commonly used in the world of swaptions. We shall cover this point in more detail later. The fourth instrument we cover is the forward swap, which is a forward contract to enter into a swap. Obviously a forward swap commits the two parties to enter into a swap, whereas a swaption gives one party the right to enter into a swap.

These types of instruments exist with underlyings other than interest rates, but we have already covered such instruments as forwards and options on currencies and equities. The interest rate derivatives market is much larger than the market for currency and equity derivatives. Interest rate derivatives are also different from currency and equity derivatives and, thus, merit special consideration. Of course, we have already covered the most important interest rate derivative, the swap. While currency and equity swaptions and forward swaps exist, the market is quite small relative to the market for interest rate swaptions and forward swaps, and we do not cover equity and currency versions of these instruments in this book.

Data are not available on the size of the market for swaptions and forward swaps, but Figure 13.1 shows the notional principal of interest rate options and FRAs as estimated by the Bank for International Settlements (http://www.bis.org) in its semiannual survey. As of December, 2005, the notional principal of interest rate options was about $27.9 trillion and the notional principal of FRAs was about $14.5 trillion.

This chapter is divided into three main sections, which deal with forward rate agreements, interest rate options, and swaptions and forward swaps.

FORWARD RATE AGREEMENTS

A forward rate agreement or FRA is similar to any type of forward contract, but the payoff is based on an interest rate, rather than the price of an asset. For example, suppose a financial manager believed that interest rates were going up and wanted to lock in a specific rate to be paid on a future loan. She could do that by taking a short position in a forward contract on a fixed-income security like a Treasury bill or bond. If rates rose, the security price would fall, the forward price would also fall, and the short position would be profitable, offsetting some of the effect of the higher borrowing rate. Alternatively, she could get a similar result using futures contracts.

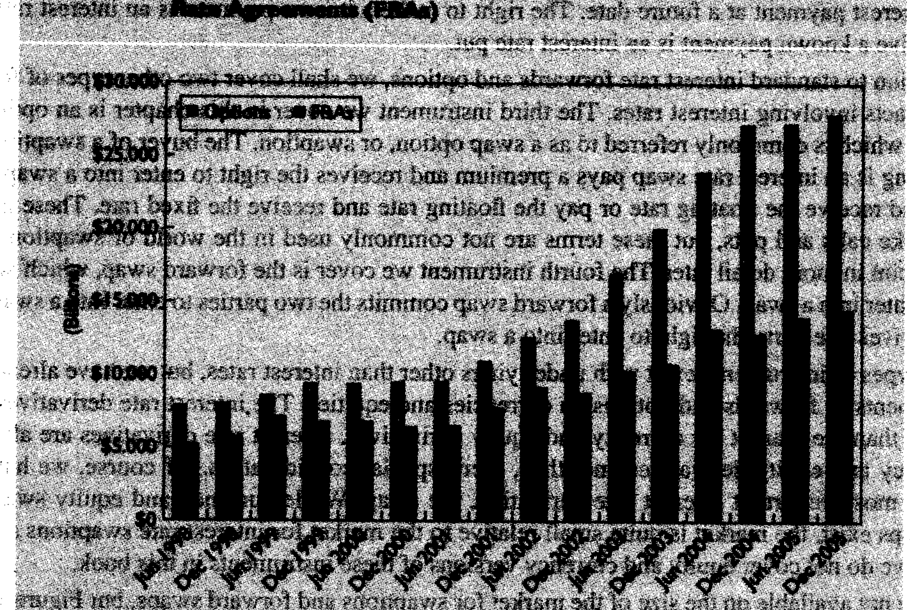
An FRA is another means of obtaining this result, and in many situations an FRA is better suited for managing this type of risk. The payoff of a forward contract on a bond is determined directly by the price of the bond and but only indirectly by the underlying interest rate. The payoff of an FRA, however, is determined *directly* by the underlying interest rate.

Structure and Use of a Typical FRA

As with swaps, FRAs are typically based on rates like LIBOR or Euribor, quoted as an annual rate. The underlying rate is for a specific term, such as 90-day LIBOR, 180-day Euribor, and so forth. Thus, the payoff is prorated by using a days/360 factor, as we did with swaps.[1] The payoff is based on the difference between the underlying rate and the rate agreed upon when the contract is established, adjusted for the number of days and multiplied by a notional principal.

[1]It is possible to use a 365-day year assumption, but FRAs are nearly always based on LIBOR, Euribor, or a similar rate, which are always based on a 360-day year assumption.

FIGURE 13.1 Notional Principal of Interest Rate Options and Forward Rate Agreements (FRAs)



Source: Bank for International Settlements (http://www.bis.org)

In addition, there is another important factor in the payoff of an FRA that we must examine carefully. Consider an FRA based on 90-day LIBOR and assume that this FRA is expiring today. As such, the parties look in the London Eurodollar market and determine the rate on a 90-day spot LIBOR instrument. Let us assume that rate is 5 percent. Remember that the rate is on a Eurodollar interbank time deposit, a loan made by one London bank to another for 90 days. When the borrowing bank takes out this loan today, it promises to pay back the principal plus 5 percent interest 90 days later. Thus, when the rate of 5 percent is determined in the London Eurodollar market, the assumption is that the interest on such a loan is paid back 90 days later.

The FRA market uses the London interbank Eurodollar market as its source of the underlying rate. Yet when the FRA expires and LIBOR is 5 percent, the FRA pays off at the 5 percent rate *today*. The Eurodollar deposit itself, which is the source of the 5 percent rate, pays off 90 days later. Consequently, to use LIBOR to determine the FRA payoff, an adjustment is required. This adjustment is to discount the FRA payment for 90 days at the 90-day rate. Because their payoffs are deferred for 90 days, interest rate swaps and options do not require this adjustment. Recall that when LIBOR is determined at the beginning of the settlement period, the swap payment based on that rate occurs at the end of the settlement period. In Chapter 12 we called this advanced set, settled in arrears. As we shall see later in this chapter, a similar procedure occurs for interest rate options. Yet, the FRA market works differently for reasons that no one seems to know.[2] So in general, an FRA on an m-day interest rate pays off at expiration but the payoff is discounted for m days at the m-day rate, which is the settlement procedure known as advanced set, advanced settled that we briefly mentioned in Chapter 12.

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[2]It is likely that the first FRAs were created in the same departments that were trading currency forwards at dealer banks. Since currency forwards make their payoffs at expiration, FRAs were probably structured in the same manner. Evidently interest rate swaps and options were not created in this manner. They were structured much more like floating-rate loans, in which the rate is determined at one point in time, the interest accrues for a period, and is paid at the end of the period, the procedure known as advanced set, settled in arrears.

Using LIBOR as the underlying rate, the general payoff of an FRA is

[redacted]

where the reference to "LIBOR" is the underlying LIBOR when the contract expires. The "Agreed upon rate" is the rate that the two parties agree on when the contract is established. Note that LIBOR appears in both the numerator and the denominator. Recall that "m/360" denotes the accrual period which in this case is actual days divided by 360.

Suppose a party takes a long position in an FRA based on 90-day LIBOR that expires in 30 days. The notional principal is $20 million. Let the rate agreed upon by the two parties be 10 percent. Thus, in 30 days the payoff to the holder of the long position will be

$$\$20{,}000{,}000\left(\frac{(\text{LIBOR}-0.10)\left(\frac{90}{360}\right)}{1+\text{LIBOR}\left(\frac{90}{360}\right)}\right).$$

Now let us consider some possible payoffs to the holder of this FRA. Suppose LIBOR at expiration is 8 percent. Then the payoff is

$$\$20{,}000{,}000\left(\frac{(0.08-0.10)\left(\frac{90}{360}\right)}{1+0.08\left(\frac{90}{360}\right)}\right)=-\$98{,}039.$$

This means that the party who is long has to pay $98,039 to the party who is short. If LIBOR at expiration is 12 percent, the payoff is

$$\$20{,}000{,}000\left(\frac{(0.12-0.10)\left(\frac{90}{360}\right)}{1+0.12\left(\frac{90}{360}\right)}\right)=\$97{,}087.$$

Hence, the party who is long receives $97,087 from the party who is short. When LIBOR at expiration is above (below) 10 percent, the holder of the long (short) position receives a positive payoff, to be paid by the counterparty. The payoff of the holder of the short position is found by changing the sign of the payoff of the holder of the long position.

The FRA market uses a distinct terminology to describe its contracts. It refers to an FRA in the form of "A × B", where A refers to the number of months until the FRA expires and B refers to the number of months, as of the contract initiation date, in the Eurodollar time deposit underlying the FRA. For example, a 6 × 9 FRA (pronounced "6 by 9") is an FRA that expires in six months with the underlying 90-day LIBOR. That is, the underlying Eurodollar time deposit matures in nine months, which is three months or 90 days after the FRA expiration. A 12 × 18 FRA expires in 12 months and the underlying is 180-day Eurodollar time deposit, which has 18 months to go before maturity when the contract is initiated.

Pricing and Valuation of FRAs

In the example above, we assume that the parties agreed on a rate of 10 percent. In this section we shall learn how that rate is determined. In addition, we shall learn how to determine the value of the FRA at a point during its life before expiration. Let F be the rate the parties agree on at the start. Let $L_0(h)$ be the spot rate for a maturity of h days, which we shall assume is the maturity date of the FRA. The underlying is m-day LIBOR. Then $L_0(h + m)$ is the spot rate for a maturity of h + m days. We assume a notional principal of $1.

One way to determine the fixed rate on an FRA is to do another set of transactions that will replicate the payoff of the FRA. Let us do the following to replicate a long position in an FRA:

- Go short a Eurodollar time deposit maturing in h + m days that pays 1 + F(m/360). Assume that we can pay off this loan at any time prior to maturity or pay another party to take over this obligation.
- Go long a Eurodollar time deposit maturing in h days that pays $1.

In other words, we borrow 1 + F(m/360) for h + m days. We lend $1 to be paid back in h days.

Now move forward to day h. The loan we owe is not due but it has a market value of

$$\frac{1+F\left(\frac{m}{360}\right)}{1+L_h(m)\left(\frac{m}{360}\right)},$$

which reflects the discounting of the payoff 1 + F(m/360) at the LIBOR rate at the expiration, which is $L_h(m)$. The minus sign reflects the fact that we owe this amount. We shall now go ahead and pay off the loan early. Since this is the value of this loan, we could technically pay it off. The loan we hold is due so we receive $1. Thus, the cash flow at this point is

$$1-\frac{1+F\left(\frac{m}{360}\right)}{1+L_h(m)\left(\frac{m}{360}\right)}.$$

Algebraic rearrangement of this equation gives

$$\frac{\left(1+L_h(m)\left(\frac{m}{360}\right)\right)-\left(1+F\left(\frac{m}{360}\right)\right)}{1+L_h(m)\left(\frac{m}{360}\right)}=\frac{(L_h(m)-F)\left(\frac{m}{360}\right)}{1+L_h(m)\left(\frac{m}{360}\right)}$$

This is the payoff of the FRA, so this strategy replicates an FRA. Thus, it must have the same value as the FRA at the start. The value of the FRA at the start is the value of the loan we made minus the value of the loan we took out, which is

$$\left(\frac{1}{1+L_0(h)\left(\frac{h}{360}\right)}\right)-\left(\frac{1+F\left(\frac{m}{360}\right)}{1+L_0(h+m)\left(\frac{h+m}{360}\right)}\right),$$

which must equal zero. Setting this to zero and solving for F gives

$$F=\left(\frac{1+L_0(h+m)\left(\frac{h+m}{360}\right)}{1+L_0(h)\left(\frac{h}{360}\right)}-1\right)\left(\frac{360}{m}\right)$$